Steel Design - LRFD
AISC Steel Manual 14th edition
Beam Limit States

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1 Moment Limit State

In steel design it is often necessary to design a beam to resist bending moments. In order to design the beam according to LRFD, ϕM_n must be determined for the trial beam selected. Once ϕM_n is determined it can then be compared to the factored moment M_u to evaluate the adequacy of the selected beam. This is often an iterative process. Hence, in order to determine ϕM_n three bending limit states must be considered:

A. Lateral Torsional Buckling (LTB).

B. Flange Local Buckling (FLB).

C. Web Local Buckling (WLB).

For *each* of the above limit states beams will, *if loaded to failure*, fail in one of three failure modes:

1. Failure by yielding (material failure), $\lambda \leq \lambda_p$.

2. Failure by inelastic buckling, $\lambda_p < \lambda \leq \lambda_r$.

3. Failure by elastic buckling, $\lambda_r < \lambda$

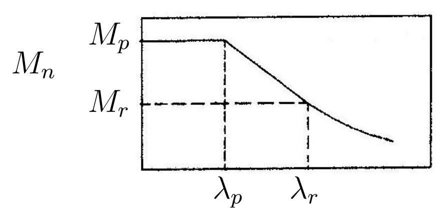

Figure I — M_n vs λ

For example, *if loaded to failure*, a beam failing according to limit state "A" will fail by failure mode 1, 2, or 3. The failure mode for the limit state "A" can be determined by calculating a *slenderness* parameter, λ. This slenderness parameter will indicate if the mode of failure occurs in the *material, inelastic, or elastic* range (see **Figure I**). Based on the mode of failure the value of ϕM_n for limit state "A"can be calculated.

This same procedure must be followed for limit state "B" and for limit state "C". By this process three values of ϕM_n, one for each limit state, are obtained. The *smallest* value of ϕM_n governs and indicates the factored nominal moment capacity of the beam which is compared to M_u.

In what follows a procedure to calculate ϕM_n for each limit state is given. For each limit state the procedure provides all of the information needed to calculate ϕM_n. In each case it is a three step process: (i)calculate the slenderness parameter, (ii)using the calculated slenderness parameter determine the failure mode, (iii)based on the failure mode calculate the value of ϕM_n. Note that the slenderness parameter, λ, and slenderness limits λ_p and λ_r are different for each of the limit states "A", "B" and "C".

A. Lateral Torsional Buckling (manual p. 16.1-47)

1. Failure by yielding (material failure) — $\lambda \leq \lambda_p$

$$\phi M_n = \phi F_y Z_x$$

2. Failure by inelastic buckling — $\lambda_p < \lambda \leq \lambda_r$

$$\phi M_n = \phi C_b \left[M_p - (M_p - M_r) \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right] \leq \phi M_p$$

3. Failure by elastic buckling — $\lambda_r < \lambda$

$$\phi M_n = \phi F_{cr} S_x \leq \phi M_p$$

where,

$\phi = 0.9 =$ strength reduction factor
$\lambda = \frac{L_b}{r_y} =$ LTB slenderness parameter
$\lambda_p = \frac{L_p}{r_y} = \frac{300}{\sqrt{F_y}} =$ Plastic LTB slenderness limit
$\lambda_r =$Inelastic LTB slenderness limit$=\frac{L_r}{r_y}$
$L_b =$ Laterally unbraced length of the beam compression flange, in
$L_p =$ Laterally unbraced length of the beam compression flange beyond which inelastic LTB begins to occur, in
$L_r =$ Laterally unbraced length of the beam compression flange beyond which elastic LTB begins to occur, in. See Manual Table 3-2, pages 3-19 to 27. See also Equation F2-6 page 16.1-48.
$F_y =$ Beam flange yield stress, ksi
$Z_x =$ Plastic modulus about beam x-axis, in^3
$r_y =$ Radius of gyration about beam y-axis, in
$C_b =$ Non-uniform beam moment, modification factor (see enclosed page regarding C_b)
$M_p = Z_x F_y =$ Plastic moment of the beam, k-in
$M_r = 0.7 F_y S_x \leq M_p$, Beam moment capacity at boundary between inelastic and elastic LTB, k-in
$F_{cr} =$ see manual pages 16.1-47 to 48 (C_b is included in the formula for F_{cr}).

Note: When possible, using the steel manual charts (pages 3-99 to 142) to find ϕM_n is simpler than using the equations above.

C_b — **Non-uniform beam moment, modification factor.** The formulas for critical elastic LTB moment were derived for a beam with constant moment stress in the compression flange. Constant moment stress in the compression flange between points of lateral support is the worst condition which may lead to a failure mode of LTB. However, in practice beams are often loaded such that a *non-constant* moment exists between points of compression flange lateral support. Hence, for situations of non-constant moment the resistance of the beam to LTB is *higher* than for a situation of constant moment. To account for this the factor C_b *increases* the value of ϕM_n when the moment is non-constant between points of compression flange lateral support (*or over the unbraced segment*). Some general rules and formulas for calculating C_b are given below.

Some general rules regarding C_b (see also manual page 16.1-302 to 308)

1. For constant moment $C_b = 1.0$
2. C_b is always greater than or equal to 1.0
3. In all situations it is conservative to use $C_b = 1.0$
4. For cantilevers or overhangs where the free end is unbraced, $C_b = 1.0$
5. In all cases **Equation I** may be used to calculated C_b
6. If the moment varies linearly between points of compression flange lateral support the simpler **Equation II** or **Table I** may be used.

Equation I

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} R_m \leq 3.0$$

where,

M_{max} = Absolute maximum moment in the unbraced segment
M_A = Absolute moment at 1/4 point of unbraced segment
M_B = Absolute moment at midpoint of unbraced segment
M_C = Absolute moment at 3/4 point of unbraced segment
R_m = 1.0 for doubly symmetric members, for other cases see manual page 16.1-305.

Equation II (only applicable for linear moment variation)

$$C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3$$

where,

M_1 = smallest absolute value end moment for the unbraced segment
M_2 = largest absolute value end moment for the unbraced segment
Note: When the moments cause single curvature in the unbraced segment the ratio of M_1/M_2 shall be input into **Equation II** as a negative number, and shall be input as a positive number when the moments cause double curvature.

Table I - Comparison of C_b for *Linear* Moment Variation

$\frac{M_1}{M_2}$	C_b **Equation I**	C_b **Equation II**
-1.00	1.00	1.00
-0.75	1.11	1.13
-0.50	1.25	1.30
-0.25	1.43	1.51
0.00	1.67	1.75
0.25	2.00	2.03
0.50	2.17	2.30
0.75	2.22	2.30
1.00	2.27	2.30

B. Flange Local Buckling (manual page 16.1-49)

1. Failure by yielding (material failure) — $\lambda \leq \lambda_p$

$$\phi M_n = \phi F_y Z_x$$

2. Failure by inelastic buckling — $\lambda_p < \lambda \leq \lambda_r$

$$\phi M_n = \phi \left[M_p - (M_p - M_r) \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p} \right) \right] \leq \phi M_p$$

3. Failure by elastic buckling — $\lambda_r < \lambda$

$$\phi M_n = \phi \frac{0.9 E k_c S_x}{\lambda^2}$$

where,

$\phi = 0.9 =$ strength reduction factor
$\lambda = \frac{b_f}{2t_f} =$ FLB slenderness parameter, may be calculated or obtained from steel manual properties table for W shapes.
$\lambda_p = 0.38\sqrt{E/F_y} =$ Plastic FLB slenderness limit for flanges of I-shaped rolled beams
$\lambda_r = 1.0\sqrt{E/F_y} =$ Inelastic FLB slenderness limit for flanges of I-shaped rolled beams
$b_f =$ Beam flange width, in
$t_f =$ Beam flange thickness, in
$M_p = Z_x F_y =$ Plastic moment of the beam, k-in
$M_r = 0.7 F_y S_x \leq M_p$, Beam moment capacity at boundary between inelastic and elastic FLB, k-in
$F_y =$ Beam flange yield stress, ksi
$Z_x =$ Plastic modulus about beam x-axis, in^3
$E = 29 \text{ x } 10^3$ ksi $=$ Modulus of elasticity for steel

S_x = section modulus about the x-axis, in^3
$k_c = \frac{4}{\sqrt{h/t_w}}$
$h = (d - 2k_{des})$, for hot rolled shapes

C. Web Local Buckling (assume steel manual beams all have compact webs, $\lambda \leq \lambda_p$)

Failure by yielding (material failure) — $\lambda \leq \lambda_p$

$$\phi M_n = \phi F_y Z_x$$

where,
$\lambda = \frac{h}{t_w}$ = Slenderness parameter for WLB, may be calculated or obtained from steel manual properties table for W shapes.
$\lambda_p = \frac{640}{\sqrt{F_y}}$ = Plastic WLB slenderness limit

2 Shear Limit State

The factored shear capacity for beams, ϕV_n, is determined as follows (manual pages 16.1-67 to 16.1-69):

$$\phi V_n = \phi 0.6 F_y A_w C_v$$

where,
$\phi = 1.0$ for webs of I-shaped members with $h/t_w \leq 2.24\sqrt{E/F_y}$
$\phi = 0.9$ for all other shapes and(or) h/t_w conditions
F_y = the yield capacity of the steel beam web
A_w = the area of the steel web calculated as $t_w d$
$h = (d - 2k_{des})$, for hot rolled shapes
d=total beam depth
k_{des} = distance from top of flange to bottom of flange fillet
t_w = thickness of the beam web
C_v = the web shear coefficient, can be taken as 1.0 for beams in the steel manual when $h/t_w \leq 2.24\sqrt{E/F_y}$, which is true for almost all current W, S, and HP shapes

In general, C_v is determined as follows:

a. For $\frac{h}{t_w} \leq 1.10\sqrt{\frac{k_v E}{F_y}}$

$$C_v = 1.0$$

b. For $1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \leq 1.37\sqrt{\frac{k_v E}{F_y}}$

$$C_v = \frac{1.10\sqrt{\frac{k_v E}{F_y}}}{\frac{h}{t_w}}$$

c. For $\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}}$

$$C_v = \frac{1.51 E k_v}{\left(\frac{h}{t_w}\right)^2 F_y}$$

where the web plate buckling coefficient, k_v, is determined as follows:

i. For webs that are not stiffened and have $h/t_w < 260$, $k_v = 5$. However, $k_v = 1.2$ for the stem of tee shapes.

ii. For webs with stiffeners,

$$\begin{aligned} k_v &= 5 + \frac{5}{(a/h)^2} \\ &= 5 \text{ when } a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2 \end{aligned}$$

where a is the clear distance between transverse stiffeners and h is as defined previously for hot rolled shapes.

3 Deflection Limit State

If deflections are large enough they may impair the functionality of a building. They may also cause damage to the interior or exterior finishes of the building. For these reasons deflection is a serviceablity limit state. Therefore, deflections are determined by using unfactored loads. The deflection criteria can be written as follows:

$$\Delta_{actual} \leq \Delta_{limit}$$

Deflection limits are usually prescribed by relevant building codes. For example, the deflection limits in the 2003 IBC are as follows:

Deflection limits for various member types and load conditions

Member type	LL	$DL + LL$	S or W
Floor Member	$\frac{L}{360}$	$\frac{L}{240}$	–
Roof Member supporting plaster ceilings	$\frac{L}{360}$	$\frac{L}{240}$	$\frac{L}{360}$
Roof Member supporting nonplaster ceilings	$\frac{L}{240}$	$\frac{L}{180}$	$\frac{L}{240}$

4 Conclusions

Beam limit states fall into two basic categories, (i) strength limit states and (ii) serviceablity limit states. Moment capacity and shear capacity are strength limit states, whereas deflection is a serviceablity limit state. Each of these limit states are important considerations for the functionality of the structure being designed. Other limit states for strength also exist such as local flange bending, local web yielding, web crippling, sidesway web buckling and compression buckling of the web, all of which are potentially caused by concentrated loads acting on webs and flanges. These additional limit states are addressed in the AISC steel manual section J10 (pages 16.1-133 to 16.1-139). These additional limit states have not been addressed in this document and should be given due consideration during the design of beams when appropriate.

It is also worth stating that this document is primarily applicable to x-x axis bending of hot rolled I-beams listed in the AISC steel manual. For other shapes, for bending about the y-y axis or the design of plate girders the reader should carefully consult the guidelines found in the manual.