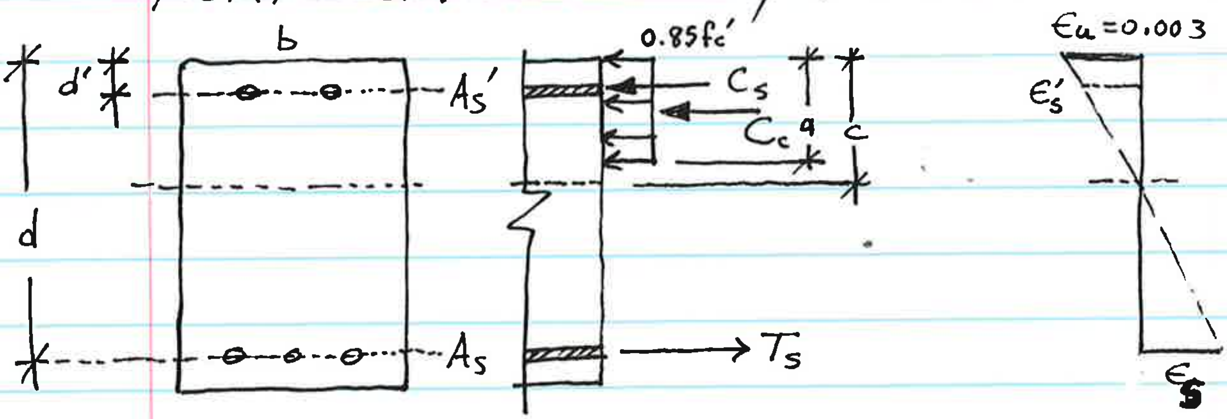
Rectangular Beams With Compression Reinforcement. "Doubly" Reinforced Beams



- By horizontal Equilibrium (assuming A_s' is above c)

$$T_s = C_s + C_c \tag{1}$$

$$\Rightarrow \quad A_s f_y = \underbrace{A_s'(f_s' - 0.85 f_c')}_{C_s} + \overbrace{0.85 f_c' \, b \, a}^{C_c} \tag{2}$$

where $$f_s' = E_s \epsilon_s' \tag{3}$$

- Strain Compatibility requires that

$$\frac{\epsilon_s}{d-c} = \frac{\epsilon_u}{c} \quad \Rightarrow \quad \epsilon_s = \epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u \tag{4}$$

and

$$\frac{\epsilon_s'}{c-d'} = \frac{\epsilon_u}{c} \quad \Rightarrow \quad \epsilon_s' = \left(\frac{c-d'}{c}\right)\epsilon_u \tag{5}$$

- Nominal Moment Capacity (once c is known)

$$M_n = C_c\left(d - \frac{a}{2}\right) + C_s(d - d') \tag{6}$$

- Determine ϕ based on ϵ_t
- Check ρ_{min} and $\epsilon_t > 0.004$ which is max. requirement.

- **Case of compression steel yielded.**

$\rightarrow f_s' = f_y$

- From (2) $\Rightarrow$ $a = \dfrac{A_s f_y - A_s'(f_s' - 0.85 f_c')}{0.85 f_c' b}$

- From (6)

(by somming moments about tensile steel)

$$M_n = 0.85 f_c' b a \left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85 f_c')(d - d')$$

- Calculate ϕ based on ϵ_t

- Check ρ_{min} and max limitation of $\epsilon_t > 0.004$

$\rho_{min} = \dfrac{200}{f_y}$ or $\dfrac{3\sqrt{f_c'}}{f_y}$ (the greater of these is ρ_{min})

- Case of Compression Steel Not Yielded

– From (2) $\Rightarrow A_s f_y = A_s'(f_s' - 0.85 f_c') + 0.85 f_c' b a$

– Using (3) and (5) $T = C_s + C_c$

$$\Longrightarrow A_s f_y = A_s'\left(E_s\left(\frac{c-d'}{c}\right)\epsilon_u - 0.85 f_c'\right) + 0.85 f_c' b \beta_1 c \qquad (*)$$

– Solve equation (*) for c by trial and error.
See text p. 144 W&M 5th

– Once c is found calculate $a = \beta_1 c$

– Calculate $f_s' = E_s\left(\frac{c-d'}{c}\right)\epsilon_u$

– Calculate M_n (by summing moments about tensile steel)

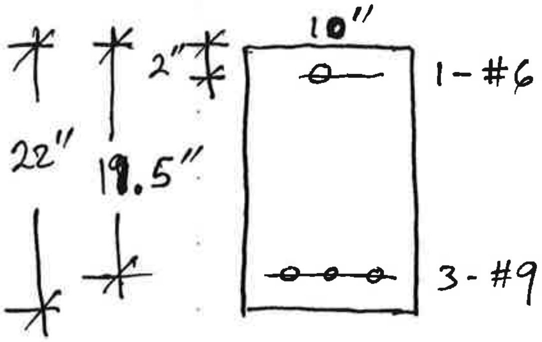
$$M_n = 0.85 f_c' b a\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d')$$

– Calculate ϕ based on $\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u$

– Check ρ_{min} and max. requirement of $\epsilon_t > 0.004$

Assuming.

Example - case of Compression steel yielding



$f_c' = 3$ ksi

$f_y = 60$ ksi

Results from computer program

$c = 7.14$

$\epsilon_t = 0.0052$

$\phi = 0.9$

$M_n = 2990$ k-in

$\phi M_n = 2691$ k-in

For Verification

Find ϕM_n

Solution: • $A_s' = 0.44 \text{ in}^2 \quad A_s = 3(1) = 3 \text{ in}^2$

• we assume compression steel has yielded (tensile steel too!)

$\Rightarrow$ from eq. (2) $\Rightarrow A_s f_y = A_s'(f_y - 0.85 f_c') + 0.85 f_c' b a$

• Solve for a

$$\rightarrow a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} = \frac{3(60) - 0.44(60 - 0.85(3))}{0.85(3)(10)} = 6.07 \text{ in}$$

• Find c

$$c = \frac{a}{\beta_1} = \frac{6.07}{0.85} = 7.14 \text{ in}$$

• Verify compression steel yielded

from (5) $\Rightarrow \epsilon_s' = \left(\frac{c - d'}{c}\right)\epsilon_u = \frac{7.14 - 2}{7.14}(0.003) = 0.00216 > \epsilon_y = \frac{f_y}{E_s} = \frac{60}{29000} = 0.00207$

ok

• Calculate M_n

from (6) $\Rightarrow M_n = 0.85 f_c' b a\left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85 f_c')(d - d')$

$$= 0.85(3)(10)(6.07)\left(19.5 - \frac{6.07}{2}\right) + 0.44(60 - 0.85(3))(19.5 - 2) = 2990 \text{ k-in}$$

• Calculate ϵ_t

from (4) $\Rightarrow \epsilon_t = \left(\frac{d - c}{c}\right)\epsilon_u = \left(\frac{19.5 - 7.14}{7.14}\right)0.003 = 0.00519 > 0.005 \Rightarrow \phi = 0.9$

Max limit satisfied $\epsilon_t > 0.004$

• Check $\rho_{min} = \frac{200}{f_y} = \frac{200}{60000} = 0.0033 \leftarrow$ governs

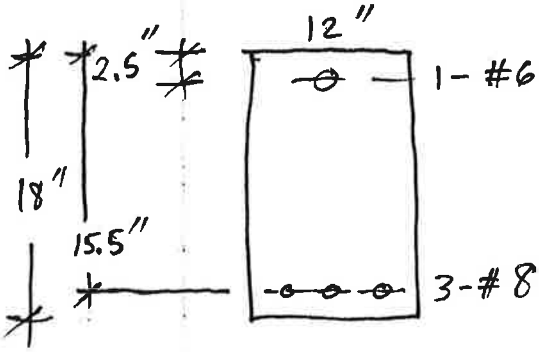
$\frac{3\sqrt{f_c'}}{f_y} = 0.0027$

$\rho = \frac{A_s}{bd} > \rho_{min}$ provided ok

• Calc. ϕM_n

$$\phi M_n = 0.9(2990) = \boxed{2691 \text{ k-in}}$$

Example Case of Compression Steel Not yielded



$f'_c = 3\,ksi$

$f_y = 60\,ksi$

Results from Computer program
$c = 4.8$ in
$\epsilon_t = 0.00688$
$\phi = 0.9$
$M_n = 1906$ k-in
$\phi M_n = 1715$ k-in
$\epsilon'_s = 0.00144$

(for Verification)

Find ϕM_n

Solution: • $A'_s = 0.44\ in^2$, $A_s = 3(0.79) = 2.37\ in^2$

• If we didn't know the steel yielded, which generally is the case we proceed by guessing a value of c. See W. & M. 5th p.144 for procedure.

• Authors suggest guess for c of between d/4 and d/3

$\Longrightarrow$ try $c = \frac{d}{3} = 15.5/3 = 5.17$ in

• Calc. $\epsilon'_s = \left(\frac{c-d'}{c}\right)\epsilon_u = \left(\frac{5.17-2.5}{5.17}\right)0.003 = -0.00155 < \epsilon_y$

• Check horizontal Equilibrium

$\Longrightarrow T = A_s f_y = 2.37(60) = 142.2$ kips

use positive value of strain

$$C_s + C_c = A_s'(E_s\epsilon_s' - 0.85f_c') + 0.85f_c' b\beta_1 c = 0.44\big(29000(0.00155) - 0.85(3)\big) + 0.85(3)(12)0.85(5.17)$$

$= 153.1$ kips difference between T and $(C_s+C_c) > 5\%$

since $T < C_s + C_c$ we need to decrease c

• New guess try $c = 4.8$ in (← cheating since I know computer results)

• Calc. $\epsilon'_s = \left(\frac{4.8-2.5}{4.8}\right)0.003 = -0.00144 < \epsilon_y$

• Check Horizontal Equilibrium

$T = 142.2$ kips

$$C_s + C_c = 0.44\big(29000(0.00144) - 0.85(3)\big) + 0.85(3)(12)(0.85)(4.8) = 142.1\ kips$$

$\Longrightarrow T \approx C_s + C_c$ good! $C = 4.8$ in is right.

- Check ϵ_t, to get ϕ

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u = \frac{15.5-4.8}{4.8}(0.003) = 0.00688 > 0.005$$

$$\Longrightarrow \phi = 0.9$$

- Calculate Nominal Moment by summing moments about tension steel.

Eq (6) $\Rightarrow$

$$M_n = 0.85\overset{f'_c}{(3)}\overset{b}{(12)}\overset{\beta_1}{0.85}\overset{c}{(4.8)}\left(\overset{d}{15.5} - \overset{a/2}{\frac{0.85(4.8)}{2}}\right) + \overset{A'_s}{0.44}\left(\overset{E_s}{29000}\overset{\epsilon'_s}{(0.00144)} - 0.85\overset{f'_c}{(3)}\right)\left(\overset{d}{15.5} - \overset{d'}{2.5}\right)$$

$$= 1905 \text{ k-in}$$

- Calculate ϕM_n

$$\phi M_n = 0.9(1905) = \boxed{1714 \text{ k-in}}$$

- Check ρ_{min}

$$\rho_{min} = \frac{200}{f_y} = 0.0033 \quad \text{or} \quad \frac{3\sqrt{3000}}{60000} = 0.00274 \Rightarrow \rho_{min} \overset{\text{governs}}{=} 0.0033$$

$$\rho_{provided} = \frac{A_s}{bd} = \frac{2.37}{12(15.5)} = 0.0127 > \rho_{min} \quad \underline{\text{ok}}!$$

- Check max. reinf. limits

$$\epsilon_t > 0.004 \quad \therefore \text{ satisfied!}$$