T- Beam Analysis (see ACI sect. 8.12)

Effective Width

1. For Symmetrical T-beams

b_{eff} must be less than $\frac{1}{4}L$

b_{eff} must be less than $b_w + 16h_f$

b_{eff} must be less than $b_w + \frac{S_c}{2} + \frac{S_c}{2}$

2. For Beams w/ slab on one side only.

$b_{eff} \leq b_w + \frac{1}{12}L$

$b_{eff} \leq b_w + 6h_f$

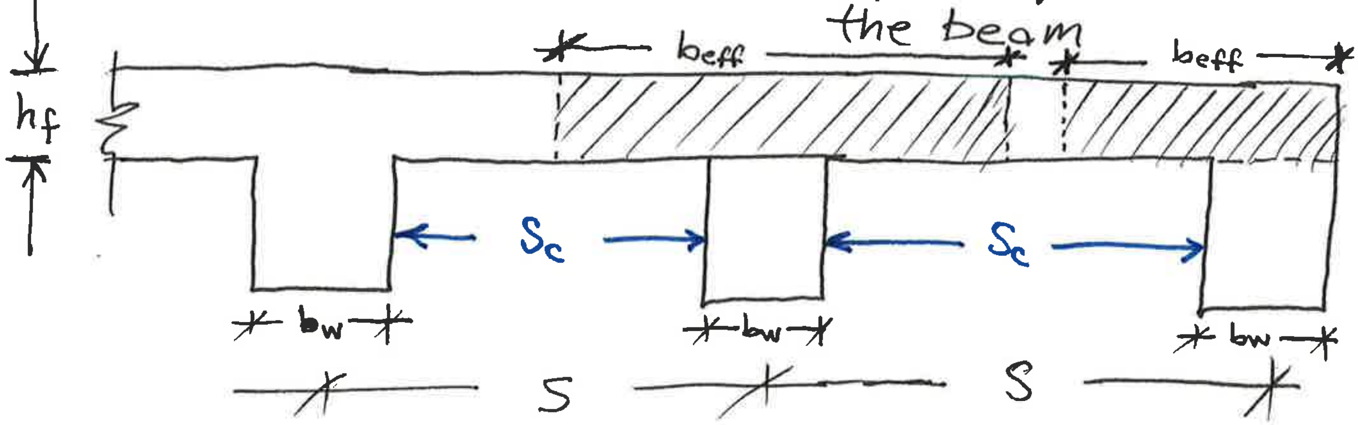
$b_{eff} \leq b_w + \frac{S_c}{2}$

3. For Isolated Beams w/ flange used for purposes of increasing compressive area.

$h_f \geq \frac{b_w}{2}$

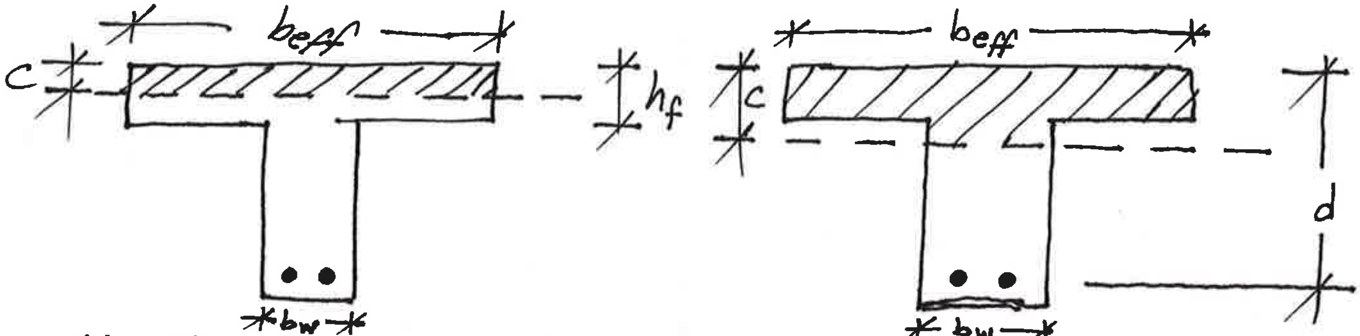
$b_{eff} \leq 4b_w$

L = Span Length of the beam



T-beam Analysis

- Two Cases Are Possible



(i) Neutral Axis Within Flange Thickness, h_f

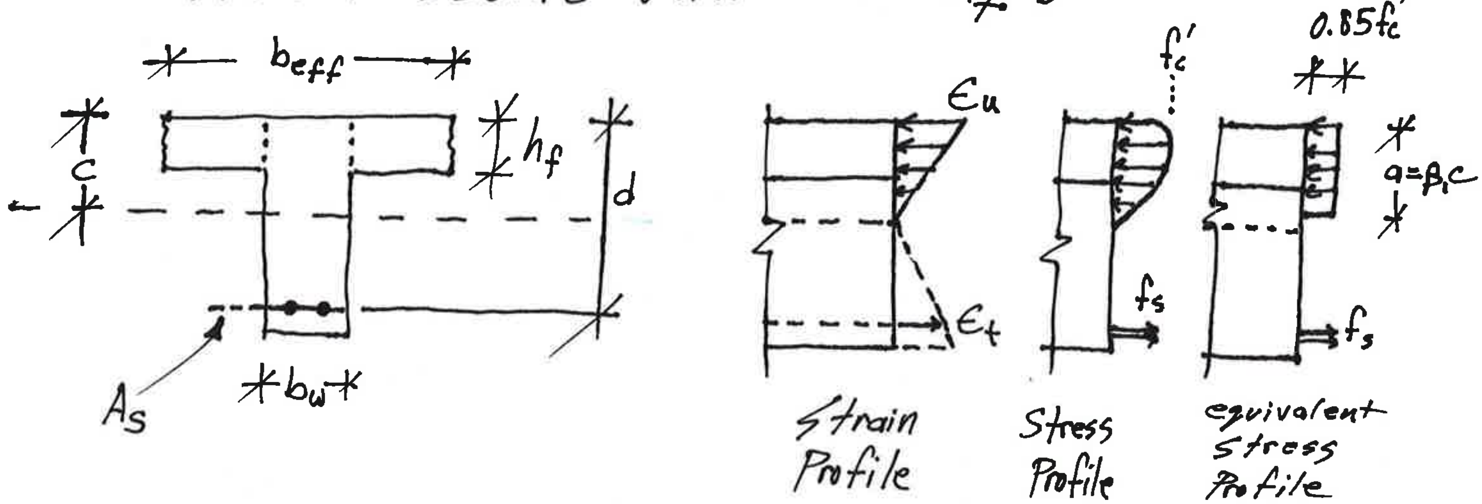
(ii) Neutral Axis Located at a distance $c > h_f$

- If case (i) occurs the beam may be analyzed exactly the same as a rectangular beam with $b = b_{eff}$.

- If case (ii) occurs we must rederive our formulas for M_n and we must find the location of c by an iterative process.

- For Case with T-beam flange in tension see ACI Sect. 10.6.6

Consider the following T-Beam And Assume that $c > h_f$.



- Consider Dividing the tensile steel into two parts.

(Part I) A_{sf} is the portion of A_s required to balance the horizontal compressive force acting on the overhanging flanges only. We assume $f_s = f_y$

$$\Rightarrow \Sigma F_x = 0; \Rightarrow \quad f_y A_{sf} = (b_{eff} - b_w) h_f (0.85 f_c')$$

$$\Longrightarrow A_{sf} = \frac{0.85 f_c' (b_{eff} - b_w) h_f}{f_y}$$

The nominal resisting moment provided by this Compressive stress and this Area of the steel is

$$M_{n_1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

(Part II) $(A_s - A_{sf})$ is the remaining portion of the steel that balances with the compressive force in the rectangular portion of the beam. Again $f_s = f_y$

$$\Rightarrow \Sigma F_x = 0; \quad f_y (A_s - A_{sf}) = 0.85 f_c' a\, b_w$$

$$\Longrightarrow a = \frac{f_y (A_s - A_{sf})}{0.85 f_c' b_w}$$

The nominal resisting moment provided by this Compressive stress and this area of steel is

$$M_{n_2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

The resulting total nominal resisting moment is the sum of the parts

$$\Longrightarrow M_n = M_{n_1} + M_{n_2} = A_s f_y\left(d - \frac{h_f}{2}\right) + (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

This moment is reduced by ϕ as per rectangular beams, and hence is dependent on ϵ_t.

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w}$$

$$c = \frac{a}{\beta_1} \qquad 0.65 \leq \phi \leq 0.9$$

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u \qquad \phi = 0.483 + 83.3\,\epsilon_t$$

- Must check ρ_{min} and max steel limit $\epsilon_t < 0.004$.

See discussion on $\rho_{min} = \frac{A_{s,min}}{b_w d}$ on text p. 154 W&M 5th.

$\Rightarrow$ For tension on bottom of T-beam

use greater value $\rightarrow$ $\begin{cases} \rho_{min} = \dfrac{A_{s,min}}{b_w d} = \dfrac{3\sqrt{f_c'}}{f_y} \\ \text{or} \\ \rho_{min} = \dfrac{200}{f_y} \end{cases}$

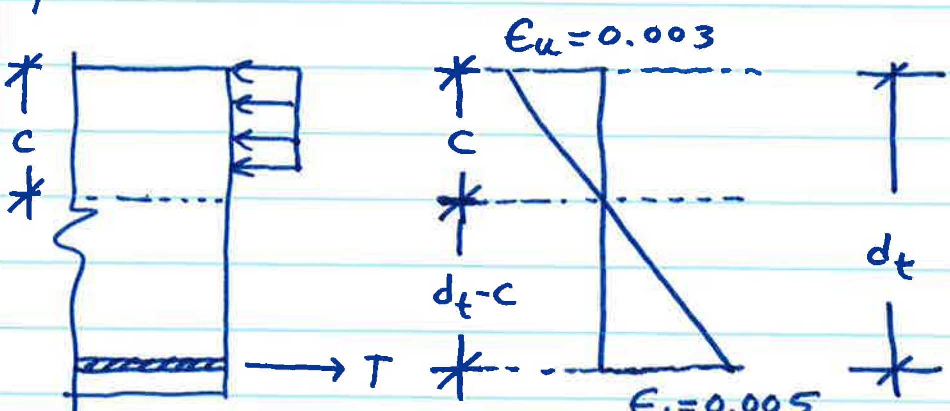
$\Rightarrow$ For continuous T-beams with flanges in tension use ρ_{min} same as above.

$\Rightarrow$ For statically determinate T-beams with flanges in tension replace b_w in eq's with the smaller of $2b_w$ or b_{eff}, but need not exceed actual flange width. (see ACI 10.5.2)

See text page 131 W. & M. 5th

Easy way to check if $\epsilon_t > 0.005$

- At tension controlled limit strain profile is



At tension control limit we want

$$\frac{0.003}{c} \le \frac{0.005}{d_t - c}$$

$$\Rightarrow 0.003 d_t - 0.003c \le 0.005c$$

$$\Rightarrow 0.003 d_t \le 0.008c$$

$$\Rightarrow c \le \frac{3}{8} d_t$$

$$\Rightarrow \frac{c}{d_t} \le \frac{3}{8}$$

$$\boxed{\text{So if } \frac{c}{d_t} \le 3/8 \text{ then } \epsilon_t \ge 0.005}$$

Problem – Case of $c < h_f$, normal beam analysis

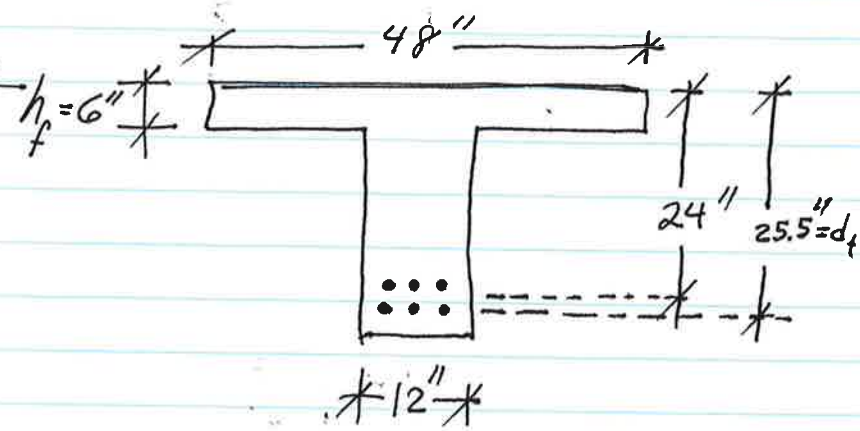
Given

A T-beam has an effective width of flange of 48 in.

$h_f = 6\text{ in} \quad b_w = 12\text{ in} \quad h = 28\text{ in}$

$d = 24\text{ in}$ (to the centroid of 6 No. 9 bars)
$f_c' = 3\text{ ksi} \quad f_y = 60\text{ ksi}$

Determine ϕM_n



Solution:

- Assume 1st $℄$ falls within depth of h_f. If this is true a regular rectangular beam analysis is necessary.

$$\longrightarrow a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6(1.0)\,60}{0.85(3)(48)} = 2.941\text{ in}$$

$$a = \beta_1 c \qquad f_c' = 3\text{ ksi} \Rightarrow \beta_1 = 0.85$$

$$\Longrightarrow c = a/\beta_1 = 2.941/0.85 = 3.46\text{ in} < h_f$$

$\therefore$ a rectangular beam analysis is ok

$$\Rightarrow \frac{c}{d_t} = \frac{3.46}{25.5} = 0.136 < 0.375 \Rightarrow \epsilon_t > 0.005\text{ in/in}$$

See text p. 131

$$\Longrightarrow \phi = 0.9$$

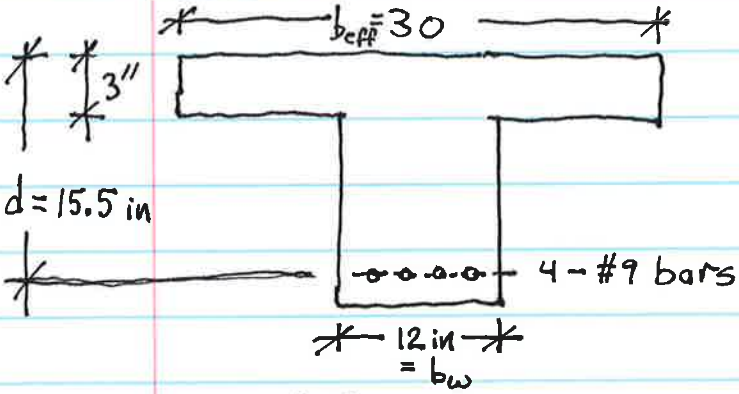
$$\longrightarrow \phi M_n = 0.9(6)\,60\left(24 - \frac{2.941}{2}\right) = 7300\text{ k-in}$$

$$= \boxed{608\text{ k-ft}}$$

- Check ρ_{min} ...

Steel Given.

Problem - Case of $c > h_f$, T-beam analysis required.



$f_y = 60\text{ksi}$

$f_c' = 3\text{ksi}$

Computer results for verification

$c = 3.93$ in

$\epsilon_t = 0.00882$

$M_n = 3342$ k-in

$\phi = 0.9$

$\phi M_n = 3008$ k-in

Find ϕM_n

Solution: $A_s = 4(1.0) = 4 \text{ in}^2$

- Check if Normal beam Analysis Required.

  Assume $c < h_f = 3"$ ← see if this is true

  $$a = \frac{A_s f_y}{0.85 f_c' b_{eff}} = \frac{4(1)(60)}{0.85(3)(30)} = 3.13 \text{ in}$$

  $$\Rightarrow c = \frac{a}{\beta_1} = \frac{3.137}{0.85} = 3.69 \text{ in} > h_f$$

  ∴ Must Do T-beam Analysis
  Can't Do Normal Beam Analysis

- Find Portion of steel Required For Equilibrium of flanges in Compression

  $$A_{sf} f_y = 0.85 f_c' h_f (b_{eff} - b_w)$$

  $$\Rightarrow A_{sf} = \frac{0.85(3)(3)(30-12)}{60} = 2.295 \text{ in}^2$$

- Find M_{n_1} = nominal moment due to flanges only

  $$M_{n_1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 2.295(60)\left(15.5 - \frac{3}{2}\right) = 1927.8 \text{ k-in}$$

- Steel Associated with web only

$$A_{sw} = A_s - A_{sf} = 4 - 2.295 = 1.705 \text{ in}^2$$

- Calculate M_{n_2} = nominal moment capacity of the web.

$$a = \frac{A_{sw} f_y}{0.85 f_c' b_w} = \frac{1.705(60)}{0.85(3)(12)} = 3.343 \text{ in}$$

$$\Rightarrow M_{n2} = A_{sw} f_y \left(d - \frac{a}{2}\right) = 1.705(60)\left(15.5 - \frac{3.343}{2}\right)$$

$$= 1414.7 \text{ k-in}$$

- Check ϵ_t

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u = \frac{15.5 - \frac{3.343}{0.85}}{\frac{3.343}{0.85}}(0.003) = 0.00882$$

$$\Rightarrow \quad \epsilon_t > 0.005 \quad \therefore \phi = 0.9$$

and max steel limit not violated ($\epsilon_t > 0.004$)

- Final Nominal Moment Capacity

$$\phi M_n = \phi(M_{n_1} + M_{n_2}) = 0.9(1927.8 + 1414.7)$$

$$\boxed{\phi M_n = 3008 \text{ k-in}}$$

same as computer results

- Check ρ_{min}

$$\rho_{min} = \frac{200}{f_y} = \frac{200}{60000} = 0.0033 \quad \text{or} \quad \rho_{min} = \frac{3\sqrt{f_c'}}{f_y} = \frac{3\sqrt{3000}}{60000} = 0.00274$$

greater value governs

$$\Rightarrow \rho_{min} = 0.0033 \leftarrow$$

$$\rho_{prov.} = \frac{A_s}{b_w d} = \frac{4}{12(15.5)} = 0.0215 > \rho_{min} \quad \underline{\text{ok}}\,!$$

Problem - case of not knowing steel area.

Suppose $M_u = 7000$ k-in
$b_{eff} = 36$ in
$h_f = 3$ in , $d = 22$ in, $b_w = 12$ in
$f_c' = 3$ ksi $f_y = 60$ksi

Determine $A_{s\,req'd}$

Solution:

- Try a Rect. Bm Analysis assume $a = h_f$
  estimate $A_{s\,req'd}$

$$\Rightarrow A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{7000}{0.9(60)(22 - \frac{3}{2})} = 6.32 \text{ in}^2$$

$$\Rightarrow a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6.32(60)}{0.85(3)(36)} = 4.13 \text{ in} > h_f = 3 \text{ in}$$

- A T-beam Analysis is Req'd.
  Start by assuming $\phi = 0.9$

$$A_{sf} = \frac{0.85 f_c' (b_{eff} - b_w) h_f}{f_y} = \frac{0.85(3)(36-12)(3)}{60} = 3.06 \text{ in}^2$$

$$\Rightarrow \phi M_{n_1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.9(3.06)60\left(22 - \frac{3}{2}\right) = 3387 \text{ k-in}$$

$$\Rightarrow \phi M_{n_2} = M_u - \phi M_{n_1} = 7000 - 3387 = 3613 \text{ k-in}$$

- Assume $a = 3.75$ in

{See page (4) alternate →

$$\Rightarrow A_s - A_{sf} = \frac{\phi M_{n_2}}{\phi f_y (d - \frac{a}{2})} = \frac{3613}{0.9(22 - 1.875)(60)} = 3.32 \text{ in}^2$$

Check $a = \frac{(A_s - A_{sf}) f_y}{0.85 f_c' b_w} = \frac{3.32(60)}{0.85(3)(12)} = 6.51 \text{ in} \neq 3.75$

No Good

New trial

- Assume $a = \frac{3.75+6.51}{2} \simeq 5.15$ in

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y(d-\frac{a}{2})} = \frac{3613}{0.9(60)(22-\frac{5.15}{2})} = 3.44 \text{ in}^2$$

Check a

$$\Longrightarrow a = \frac{3.44(60)}{0.85(3)(12)} = 6.75 \text{ in} \neq 5.15 \text{ in}$$

No Good

- Assume $a = 7$ in

$$A_s - A_{sf} = \frac{3613}{0.9(60)(22-3.5)} = 3.62 \text{ in}^2$$

Check a

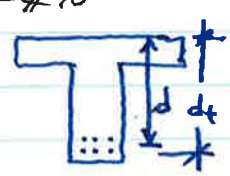
$$a = \frac{3.62(60)}{0.85(3)(12)} = 7.1 \text{ in} \simeq 7 \text{ in}$$

- Calculate req'd As

$$\Longrightarrow A_s = (A_s - A_{sf}) + A_{sf}$$
$$= 3.62 + 3.06 = 6.68 \text{ in}^2$$

- Check that net tensile strain > 0.005 met so $\phi = 0.9$

- Calculate actual "a" based on steel used assuming $d = 22$ in is to a centroid of a steel group consisting of two layers of 3-#10 bars per layer. $\Longrightarrow A_s = 7.62 \text{ in}^2$



$$\Longrightarrow a = \frac{(7.62-3.06)60}{0.85(3)(12)} = 8.94 \text{ in}$$

$$\Longrightarrow \frac{c}{d_t} = \frac{8.94/0.85}{(22+1.5)} = 0.448 \text{ in} > \frac{3}{8}$$

this is greater than max allowed
No Good

↑ to outermost tension steel

Try 3-#10 and 3-#9

$\Longrightarrow A_s = 3(1.27) + 3(1.0) = 6.81 \text{ in}^2$

$\Longrightarrow a = \dfrac{(6.81 - 3.06)60}{0.85(3)(12)} = 7.35 \text{ in}$

$\Longrightarrow \dfrac{c}{d_t} \approx \dfrac{7.35/0.85}{(22+1.5)} = 0.368 \text{ in} < 0.375$

$\longrightarrow \epsilon_t > 0.005$

To see this $\quad \epsilon_t = \left[\dfrac{0.85\beta_1 f'_c}{\left(\dfrac{A_s - A_{sf}}{b_w d_t}\right) f_y} - 1\right] 0.003 = \left[\dfrac{0.85(0.85)3}{\left(\dfrac{6.81-3.06}{12(23)}\right)60} - 1\right] 0.003 = 0.00515$

Hence $\phi = 0.9$

$\longrightarrow \phi M_n = \phi M_{n_1} + \phi M_{n_2}$

$= 3387 + 0.9(6.81 - 3.06)60\left(22 - \dfrac{7.35}{2}\right)$

$= 3387 + 3710 \text{ K-in}$

$= 7097 \text{ k-in} = 7100 \text{ k-in}$

$\longrightarrow \phi M_n > M_u$ ok

- Need to check ρ_{min}

Alternate way to do as noted on p. 1/4

Then starting w/ $\phi M_{n2} = 3613$ k-in $= 301$ k-ft

$$\rightarrow (A_s - A_{sf}) \cong \frac{\phi M_{n2}}{4d} = \frac{301}{4(22)} = 3.42 \text{ in}^2$$

$$a = \frac{3.42(60)}{0.85(3)(12)} = 6.71$$

$$\rightarrow \phi M_{n2_{trial}} = 0.9(3.42)60\left(22 - \frac{6.71}{2}\right) = 3443 \text{ k-in}$$

$< \phi M_{n2_{required}}$

No Good

$$\text{Try } A_s - A_{sf} = \frac{3613}{3443}(3.42) = 3.59 \text{ in}^2$$

$$\Longrightarrow \text{use } 3.6 \text{ in}^2 \quad a = \frac{3.6}{3.42}(6.71) = 7.06 \text{ in}$$

$$\Longrightarrow \phi M_{n2} = 0.9(3.6)(60)\left(22 - \frac{7.06}{2}\right) = 3591 \text{ k-in}$$

$\approx \phi M_{n2_{req'd}}$ <u>ok</u>

proceeds as before.