* Rectangular Reinforced Concrete Beam Design *

Checks

- Correct β_1 value
- Correct ϕ based on value of max steel strain
- Minimum ρ per code
- Max. ρ per code
- $\phi M_n > M_u$
- Adequate C/r Cover

Based on Nilson et. al.

Cases – Examples

(i)	Have M_u Have b, h, f_c', f_y Determine $A_{s\,req'd}$, ϕM_n, ρ_{min}, ρ_{max}, d
(ii)	Have M_u, f_c', f_y Determine b, d, h, $A_{s\,req'd}$, ϕM_n, ρ_{min}, ρ_{max}
(iii)	Have b, h, d, A_s, f_c', f_y Determine ϕM_n
(iv)	Have M_u, $\rho_{optimum} \approx 0.6\rho_{max}$ Determine b, d, h, A_s, ϕM_n, ρ_{min}, ρ_{max}

Example - Case (i)

Given a factored moment of 2600 K-in and a prescribed beam size of $b = 14$ in and $h = 24$ in, determine ϕ, ϕM_n, $A_{s\,req'd}$, ρ_{min}, ρ_{max}, d

Assume the beam is a beam inside a building.
Assume $f'_c = 4000$ psi, $f_y = 60000$ psi (steel), $\epsilon_u = 0.003$ in/in (concrete)

Solution:

- Determine d

Since Beam is inside building the clear cover required for the steel is $1\frac{1}{2}$ inches per ACI Code 7.7.1(c)

$$\Longrightarrow d = h - 1\tfrac{1}{2}'' - \tfrac{1}{2} d_b \qquad \text{assume a } 1''\phi \text{ bar}$$

$$\Longrightarrow d = 24 - 1.5 - \tfrac{1}{2}(1) = 22 \text{ inches}$$

- Calculate ρ_{max}, ρ_{min}

Text Nilson p. 79 per Table 3.1

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85(0.85)\frac{(4)}{(60)} \cdot \frac{0.003}{0.003 + 0.004}$$

$$\Longrightarrow \rho_{max} = 0.02064 \quad \text{(see Table A.4 p. 737, Nilson)}$$

$$\rho_{min} = \frac{200}{f_y} \text{ or } \frac{3\sqrt{f'_c}}{f_y} \quad \text{The greater is req'd} \Rightarrow \rho_{min} = 0.0033$$

- Estimate A_s (use Time Saving Design Aids (TSDA) suggestion)

in k-ft; See 12 converting it to k-ft

$$A_s \cong \frac{M_u}{4d} = \frac{2600}{4(22)12} = 2.46 \text{ in}^2$$

in.

or

p. 738 Table A.5a

$$R \cong \frac{M_u}{\phi b d^2} = \frac{2600}{0.9(14)(22)^2} = 0.4263 \text{ ksi} \Rightarrow \rho = 0.0075$$

$$\Longrightarrow A_s = \rho b d = 0.0075(14)(22) = 2.31 \text{ in}^2$$

$$\text{Try } A_s = 3(0.79) = 2.37 \text{ in}^2 \qquad \text{3-\#8 bars}$$

$$\longrightarrow \rho = \frac{2.37}{14(22)} = 0.007695$$

- Determine ϕ

  for steel strain $\epsilon_t \geq 0.005$ $\quad \phi = 0.9$

  Calculate $\epsilon_t = 0.003\left(\frac{0.85\,\beta_1 f_c'}{\rho \quad f_y} - 1\right)$ see Table A.5a

$$\Longrightarrow \epsilon_t = 0.003\left(\frac{0.85(0.85)4}{0.00759(60)} - 1\right) = 0.0158 > 0.005$$

$$\Longrightarrow \phi = 0.9$$

- Calculate ϕM_n

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.37(60)}{0.85(4)(14)} = 2.987 \text{ in}$$

$$\Longrightarrow \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

$$\Longrightarrow \phi M_n = 0.9(2.37)(60)\left(22 - \frac{2.987}{2}\right) = 2624 \text{ k-in}$$

$$\Longrightarrow \phi M_n > M_u \quad \underline{\underline{ok}}$$

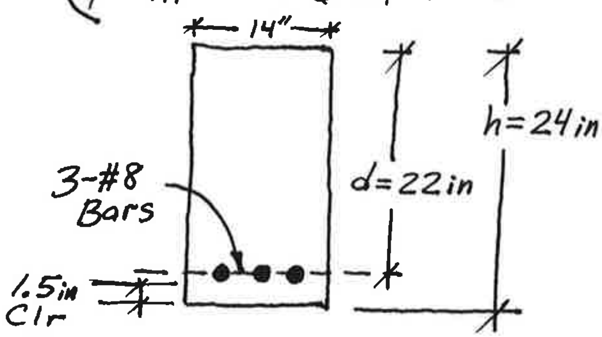
Summary

$\rho_{min} = 0.0033$ $\quad \rho_{max} = 0.02064$ $\quad \rho_{min} < \rho_{provided} \leq \rho_{max}$ $\underline{\underline{ok}}$

$\phi = 0.9$ $\quad d = 22$ in

$A_s = 2.37 \text{ in}^2$ (3-#8 bars) Note: #8 bars are 1"ϕ original assumption $\underline{ok}$!

$\phi M_n = 2624$ k-in



Note - Should have used 21.5 in for d to provide room for stirrups, and still maintain sufficient cover.

$d = 21.5\text{in} \Rightarrow \phi M_n = 2560 \text{ k-in} \approx M_u$ w/in 2% $\underline{ok}$

Example Case (ii)

Given $M_u = 3600$ K-in

Determine $b, d, h, A_{s\,req'd}, \phi M_n, \rho_{min}, \rho_{max}$

Assume $f'_c = 3000$ psi $f_y = 60000$ psi steel, $\epsilon_u = 0.003$ in/in

$\Longrightarrow \beta_1 = 0.85$

Both ends of beam continuous, beam span $\ell = 20$ ft

Assume Beam Inside a Bldg.

Solution:

- Determine h Per ACI Table 9.5(a) $h_{min} = \dfrac{\ell}{21} = \dfrac{20(12)}{21} = 11.4$ in

$\longrightarrow$ Use $h = 14$ inches

- Determine d

Use $d = h - 2.5 = 11.5$ in

- Estimate b use (TSDA) M_u in K-ft b, d in inches

$$b \approx \frac{20 M_u}{d^2} = \frac{20(3600/12)}{11.5^2} = \underline{45.4 \text{ in}}$$

quite large

- Revise Beam size

Let's try a bigger h. Try $h = 24$ inches

$\Longrightarrow d = 21.5$ in

$$b = \frac{20(3600/12)}{(21.5)^2} = 12.98 \text{ in} \Rightarrow \text{use } 14 \text{ in.}$$

- Determine $A_{s\,req'd}$

Estimate using (TSDA) $\Rightarrow A_{s\,req'd} \approx \dfrac{3600/12}{4(21.5)} = 3.49 \text{ in}^2$

Try 5 - #8 bars $\Rightarrow A_s = 5(0.79) = 3.95 \text{ in}^2$

Allowed per Table 2 of TSDA $\Longrightarrow \rho = \dfrac{3.95}{14(21.5)} = 0.01312$

- Determine ρ_{min}, ρ_{max} Use Table A.4

$\rho_{min} = 0.0033$

$\rho_{max} = 0.0155$ $\rho_{min} < \rho < \rho_{max}$ <u>ok</u>

- Determine ϕ, Notice $\rho < \rho_{\epsilon_t = 0.005}$

  Per table A.4 (p. 737 Nilson)

  This implies that $\epsilon_t > 0.005$ for the quantity of steel we have selected.

  Hence $\phi = 0.9$

- Determine ϕM_n

  From Table A.5a $R \approx 661\ psi = 0.661\ ksi$

$\Longrightarrow \phi M_n = \phi R b d^2$

$\Longrightarrow \phi M_n = 0.9(0.661)(14)(21.5)^2 = 3850\ K\text{-}in$

$M_u = 3600\ K\text{-}in$

$\Longrightarrow \phi M_n > M_u$ ok

Summary

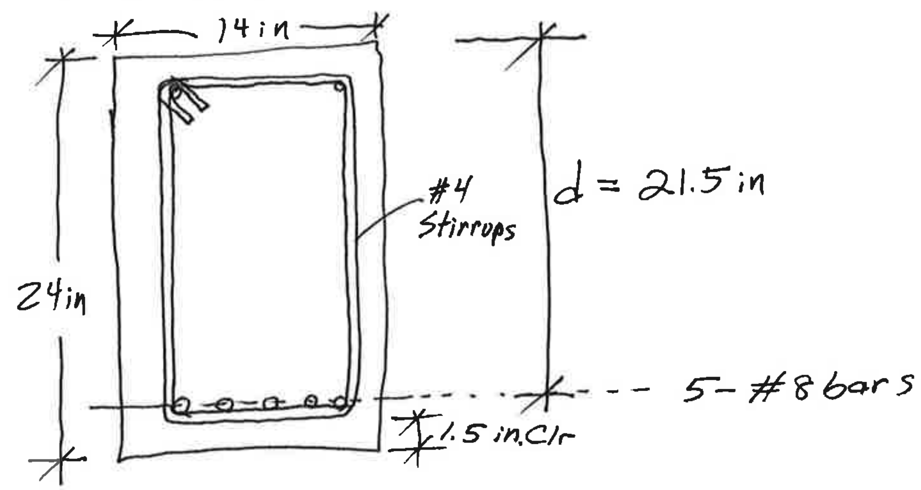
$b = 14\ in$ $\qquad \rho_{min} = 0.0033$

$d = 21.5\ in$ $\qquad \rho_{max} = 0.0155$

$h = 24\ in$

$A_s = 3.95\ in^2$ (5-#8 bars) — works for our $d = h - 2.5$ assumption

$\phi M_n = 3850\ K\text{-}in$



Example – Case (iii)

Given $b = 14$ in $d = 18$ in $h = 21$ in

$f'_c = 4$ ksi $f_y = 60$ ksi $\epsilon_u = 0.003$ in/in

$A_s = \boxed{4 - \#10 \text{ bars}}$

Determine ϕM_n

Solution:

- Determine ρ $A_{s_{bar}} = 1.27 \ in^2/bar$ for one # 10 bar

$$\rho = \frac{A_s}{bd} = \frac{4(1.27)}{14(18)} = \frac{5.08}{14(18)} = 0.02016$$

Table A.4 Nilson P.737

$$\left\{ \begin{array}{l} \rho_{max} = 0.0206 \implies \rho < \rho_{max} \ \underline{ok} \\ \rho_{\epsilon_{t=0.005}} = 0.0181 < \rho \ \text{ this implies } \epsilon_t < 0.005 \end{array} \right.$$

- Determine ϕ $f'_c = 4$ ksi $\Rightarrow \beta_1 = 0.85$

$$\epsilon_t = 0.003\left(\frac{0.85(0.85)4}{0.02016(60)} - 1\right) = 0.004168$$

$$\implies \phi = 0.483 + 83.3(0.004168) = 0.83$$

- Determine R $\rho \approx 0.020$

Table A.5a $\implies R = 0.988$ ksi

- Determine ϕM_n

$$\phi M_n = \phi R b d^2 = 0.83(0.988)(14)(18)^2 = 3720 \text{ k-in}$$
$$= 310 \text{ k-ft}$$

$$\boxed{\phi M_n = 3720 \text{ k-in} = 310 \text{ k-ft}}$$

Example - Case (iv)

Given $M_u = 800$ k-ft $f'_c = 5000$ psi $f_y = 60$ ksi

And we want an optimum $\rho_{opt} \approx 0.6 \rho_{max}$

Assume beam inside of bldg.

Determine $b, d, h, A_s, \phi M_n, \rho_{min}, \rho_{max}$

Solution:

- Determine b & d

Assume $b \cong \frac{1}{2} d$

M_u in k-ft, d in inches

$\Rightarrow$ Using (TSDA) $\frac{1}{2} d\, d^2 \approx 20 M_u$

$\Rightarrow d = (2(20)(800))^{1/3} = 31.75$ in Use 32 inches

$\Rightarrow b = 16$ inches.

- Determine A_s

Table A.4 Nilson p. 737

$\rho_{opt} \approx 0.6 \rho_{max} = 0.6(0.0243) = 0.0146$

$\Rightarrow A_{s_{req'd}} = 0.0146(16)(32) = 7.48 \text{ in}^2$

- Determine bar size and quantity

Table 2 p.455 of TSDA suggests that we need an 18 inch wide beam to get the necessary steel using 5 - #11 bars.

$\Rightarrow$ Use $b = 18$ inches

$\Rightarrow$ Use 5 - #11 bars $\Rightarrow A_s = 5(1.56) = 7.8 \text{ in}^2$

$\Rightarrow \rho = \dfrac{7.8}{18(32)} = 0.01354$

- Determine ρ_{min}, ρ_{max} use Table A.4 Nilson

$\rho_{min} = 0.0035$

$\rho_{max} = 0.0243$ $\Rightarrow \rho_{min} < \rho < \rho_{max}$ ok

- Determine ϕ

$$\rho_{\epsilon_t=0.005} = 0.0213 > \rho = 0.01354$$

this implies that $\epsilon_t > 0.005$

$$\Longrightarrow \phi = 0.9$$

- Determine R use Table A.5a Nilson p. 738

$$\longrightarrow R = 0.733 \text{ ksi}$$

- Determine ϕM_n

$$\phi M_n = \phi R b d^2 = 0.9(0.733)(18)(32)^2$$

$$\longrightarrow \phi M_n = 12160 \text{ k-in}$$
$$= 1013 \text{ k-ft}$$

$$\Longrightarrow \phi M_n > M_u \quad \underline{ok}$$ (5- #10 bars would work, Should revise but for now leave 5 #11 bars)

- Determine h

$h = d + \text{stirrup } \phi + \frac{1}{2} d_b + 1.5'' = 32 + 0.5 + \frac{1}{2}(1.41) + 1.5 = 34.7 \text{ in} \approx 36 \text{ in}$

- Summary

$b = 18$ in

$d = 32$ in

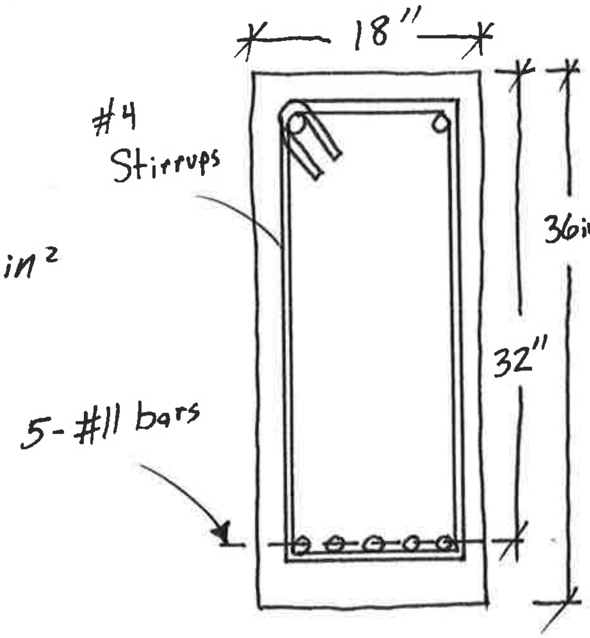
$h = 36$ in

$A_s = \boxed{5\text{- \#11 bars}} = 7.8 \text{ in}^2$

$\phi M_n = 1013$ k-ft

$\rho_{min} = 0.0035$

$\rho_{max} = 0.0243$



Discussion of Examples

(1.) The solutions given are a solution among many. There is no single Correct answer. The answer you provide will likely be influenced by your particular design's constraints.

(2) The solutions often calculate the same quantity in a variety of ways. For instance ϕM_n can be determined by Several routes:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$
$$\phi M_n = \phi R b d^2 .$$

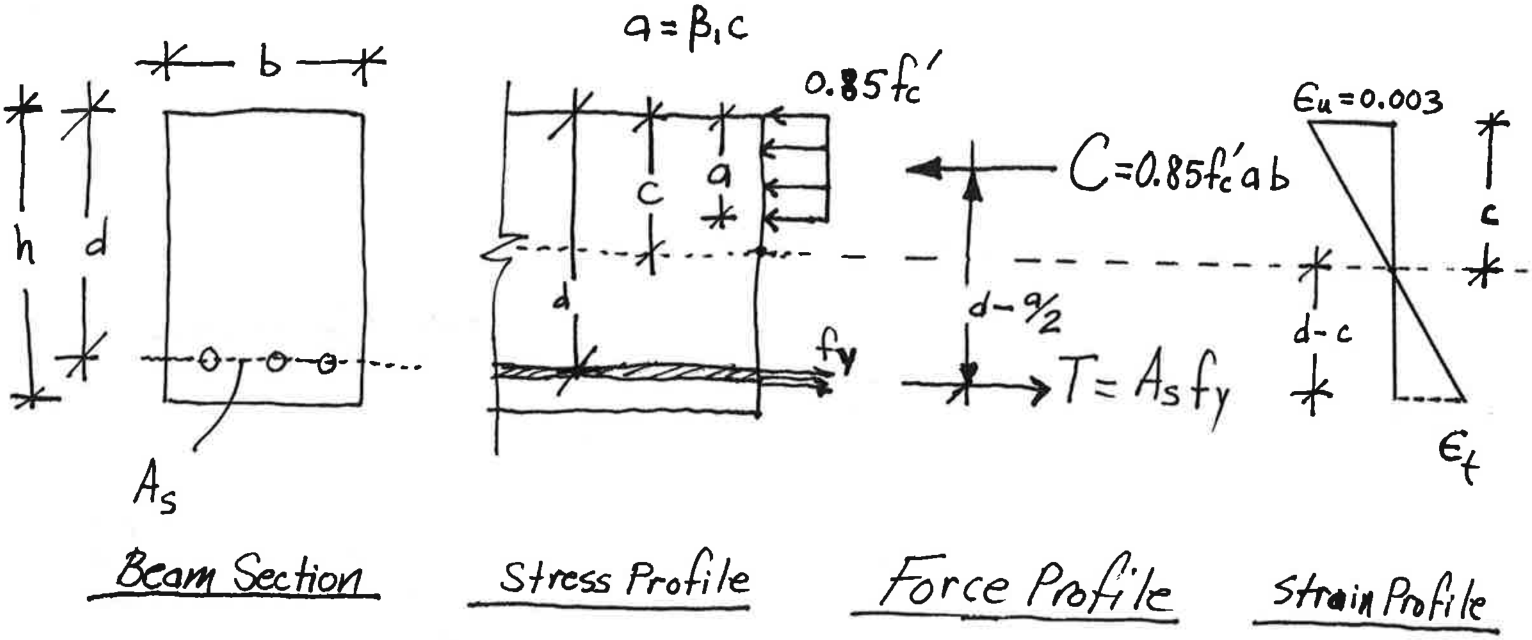
Tables or formulas can be used to get a variety of variables. For instance ρ_{min}, ρ_{max} can be calculated or obtained from Tables.

The solutions presented deliberately illustrate a variety of ways to calculate or "look up" quantities that you need. Do what works best for you.

(3) Nilson is the Author of your textbook

TSDA = Time Saving Design Aid paper

Summary



- **β_1 Factor**

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000}{1000}$$ with the restriction $0.65 \leq \beta_1 \leq 0.85$

- **ρ_{min}** (see also ACI 10.5)

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

$$\rho = \frac{A_s}{bd}$$

Sometimes helpful
$$\rho = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

- **ρ_{max}**

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$ based on ACI 10.3.5

- **ϕM_n**

$$a = \frac{A_s f_y}{0.85 f_c' b}, \quad M_n = A_s f_y \left(d - \frac{a}{2}\right) = R b d^2$$

see Nilson p. 754
$$R = \omega f_c' (1 - 0.59\omega)$$
$$\omega = \rho \frac{f_y}{f_c'}$$

$$\phi = 0.483 + 83.3 \epsilon_t, \quad 0.65 \leq \phi \leq 0.9$$

- **ϵ_t**

$$\epsilon_t = \left(\frac{d - c}{c}\right) \epsilon_u$$

TSDA – Approximations

$A_{s\,req'd} \approx \frac{M_u}{4d}$ ← M_u in k-ft, d in inches, $\quad b \approx \frac{20 M_u}{d^2}, \quad b \approx \frac{1}{2} h$

R.C. Beams and Columns – Clear Cover Design Aid
by Professor Yaw (Winter 2004)

Description: A table is presented below which lists the ACI code required distance from <u>center</u> of longitudinal bar to edge of concrete. These distances are calculated based on the required ACI clear cover requirements and the assumptions listed below. The table below will help simplify the commonly arising question of whether or not sufficient clear cover has been provided. Given the table below one can, for a particular longitudinal bar size, immediately look up the required center of bar to face of concrete distance to satisfy ACI clear cover requirements.

<u>Assumptions:</u>

- The table is per ACI 318-02 Section 7.7.1.
- The table is for Non-Prestressed Reinforced Concrete Beams or Columns.
- For #4 to #6 longitudinal bars, #3 stirrups are assumed.
- For #7 to #18 longitudinal bars, #4 stirrups are assumed.
- All calculated values of <u>C</u>enter of <u>L</u>ongitudinal <u>B</u>ar <u>T</u>o <u>F</u>ace of <u>C</u>oncrete <u>D</u>istance (CLBTFCD) are rounded up to the nearest 1/8 inch. CLBTFCD is calculated based on the following formula:

$$\text{CLBTFCD} = (\text{Min. Clr Cover Req'd}) + (\text{Stirrup Diameter}) + (\tfrac{1}{2}\, d_b)$$

<u>Center of Longitudinal Bar to Face of Concrete Distance (CLBTFCD)</u>

		For Beams or Columns					
Long. Bar Size		Cast Against Earth and Permanently Exposed to Earth.		Concrete Exposed to Earth or Weather		Concrete Not Exposed To Weather	
Size	(d_b inches)	Min. Clr Req'd	CLBTFCD(in)	Min. Clr Req'd	CLBTFCD(in)	Min. Clr Req'd	CLBTFCD(in)
#4	0.500	3.0	3.625	1.5	2.125	1.5	2.125
#5	0.625	3.0	3.750	1.5	2.250	1.5	2.250
#6	0.750	3.0	3.750	2.0	2.750	1.5	2.250
#7	0.875	3.0	4.000	2.0	3.000	1.5	2.500
#8	1.000	3.0	4.000	2.0	3.000	1.5	2.500
#9	1.128	3.0	4.125	2.0	3.125	1.5	2.625
#10	1.270	3.0	4.250	2.0	3.250	1.5	2.750
#11	1.410	3.0	4.250	2.0	3.250	1.5	2.750
#14	1.693	3.0	4.375	2.0	3.375	1.5	2.875
#18	2.257	3.0	4.750	2.0	3.750	1.5	3.250