Timesaving Design Aids for Reinforced Concrete

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Abstract

Tools and methods are presented to design safe, economical concrete buildings in less time. Specifically addressed is design and detailing of non-prestressed, reinforced concrete structural members subjected to gravity and lateral loads. All methods conform to the provisions in *Building Code Requirements for Structural Concrete (318-99) and Commentary (318R-99).*

Introduction

As schedules become tighter and codes and standards become more complex, engineers need tools and design aids to assist them in producing safe, economical structures in the shortest time possible. The information provided in this paper provides ways to reduce the design and detailing time required for non-prestressed concrete beams, one-way slabs, two-way slabs, columns, and walls.

The design aids presented here—which can be utilized in preliminary and final design stages or to verify computer output—conform to the provisions of ACI 318-99 (ACI, 1999). All referenced section numbers and notation are in ACI 318-99. These design methods and aids are an attempt to satisfy the requirements of this standard in the simplest and quickest ways possible.

It is assumed that a structural analysis has been performed—which includes gravity and lateral (wind and/or seismic) forces—and that factored reactions have been calculated for the members. Design tools are provided to assist in determining member size and reinforcement based on the factored load combinations.

Beams and One-way Slabs

The member depth is typically determined first to ensure that the deflection requirements of Section 9.5 are satisfied. For non-prestressed beams and one-way slabs that are not supporting or attached to partitions or other construction likely to be damaged by large deflections, the minimum thickness (h) is given in Table 9.5(a). For the case of one end continuous, normal weight concrete, and Grade 60 reinforcement, the minimum thickness is $\ell/24$ for solid one-way slabs and $\ell/18.5$ for beams or ribbed one-way slabs, where the span length (ℓ) is in inches.

Deflection problems can also be minimized by judiciously choosing the tension reinforcement ratio (ρ) in the positive moment region. Deflection problems are rarely encountered in beams with $\rho = 0.5\rho_{max}$.

Consider the following strength equation that must be satisfied at all sections:

$$M_u \le \phi M_n = \phi A_s f_y (d - a/2) = \phi \rho b d f_y (d - a/2)$$

where $a = A_s f_y / 0.85 f'_c b = \rho d f_y / 0.85 f'_c b$. Assuming 4,000 psi concrete and Grade 60 reinforcement, $\rho = 0.5\rho_{max} = 0.0107$ and the above equation becomes:

$$bd^2 = 22.9 M_u \approx 20 M_u$$

where, for convenience, b and d are in inches and M_u is the governing factored moment in ft-kips.

Any combination of b and d can be determined from this equation, with the only restriction that the final depth (h) selected must satisfy serviceability requirements. Thus, h can be determined from Table 9.5(a) and d can be taken as h – 2.5 inches for beams with one layer of steel and h – 1.25 inches for joists and slabs. The above sizing equation can then be solved for the width (b). Similar equations can easily be derived for other materials.

Since one-way slabs are usually designed using a one-foot strip, the above sizing equation simplifies to:

$$d = 1.3\sqrt{M_u}$$

The following should be considered when sizing beams and one-way slabs for economy:

- Use whole inches for beam dimensions; slabs may be specified in ½-in. increments.
- Use constant beam size from span to span and vary reinforcement as required.
- Use wide, flat beams (same depth as joist system when applicable) rather than narrow deep beams.
- Use beam width equal to or greater than column width.
- Repeat the same member sizes wherever possible in the building.

Following these guidelines results in economical formwork, which generally leads to the most economical structure.

For beams in special moment frames, which are required in areas of high seismic risk, the geometric constraints in Section 21.3.1 must also be satisfied.

Once the member size has been established, the negative and positive reinforcement must be determined along the span. The above strength equation can be rewritten in the following form:

$$R_n = \frac{M_u}{\phi b d^2} = \rho f_y \left[1 - \frac{0.5\rho f_y}{0.85 f'_c}\right]$$

The relationship between ρ and R_n for Grade 60 reinforcement and various concrete strengths is shown in Figure 1. The relationship between ρ and R_n is approximately linear up to about $2\rho_{max}/3$. Therefore, the term $f_y\left[1 - \frac{0.5\rho f_y}{0.85 f'_c}\right]$ is a constant up to that point, and the relationship can be described as follows:

$$\frac{M_u}{\phi b d^2} = \rho \times \text{constant} \quad \text{or} \quad A_s = \frac{M_u}{\phi d(\text{constant})}$$

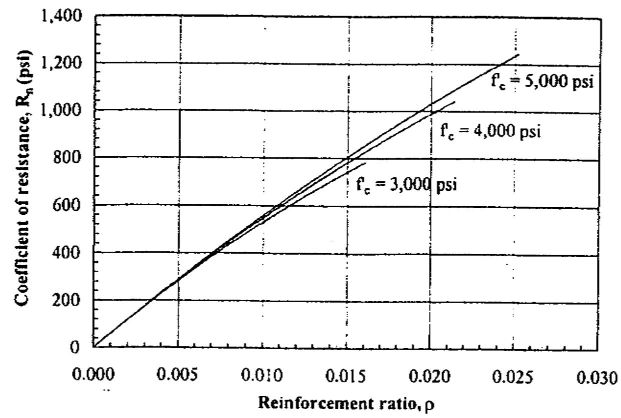


Figure 1. Strength curves for Grade 60 reinforcement.

For f'_c = 4,000 psi, f_y = 60,000 psi, and $\rho = (2/3)\rho_{max}$, the constant for the linear approximation is 4.37. Therefore,

$$A_s = \frac{M_u}{\phi d(\text{constant})} = \frac{M_u}{0.9 \times d \times 4.37} = \frac{M_u}{3.93d} \approx \frac{M_u}{4d}$$

where M_u is in ft-kips, d is in inches, and A_s is in square inches.

This equation can be used to quickly determine the required tension reinforcement at any section of a rectangular beam or slab subjected to M_u. For all values of $\rho < 2\rho_{max}/3$, it gives slightly conservative results. The maximum deviation in A_s is less than ±10% at the minimum and maximum permitted steel ratios. For members with ρ in the range of 1% to 1.5%, the error is less than 3%. Similar calculations show that this equation can also be used for members with 3,000 psi and 5,000 psi concrete.

Once the required area of steel is computed, the size and number of reinforcing bars must be selected. The minimum and maximum number of reinforcing bars permitted in a cross-section are a function of cover and spacing requirements given in Sections 7.6.1 and 3.3.2 (minimum spacing for concrete placement), Section 7.7.1 (minimum cover for protection of reinforcement), and Section 10.6 (maximum spacing for control of flexural cracking).

The maximum spacing of reinforcing bars is limited to the value given by Equation (10-5) in Section 10.6.4. The following equation can be used to determine the minimum number of bars n_{min} required in a single layer (see Figure 2):

$$n_{min} = \frac{b_w - 2(c_c + 0.5d_b)}{s} + 1$$

where, from Equation (10-5),

$$s = \frac{540}{f_s} - 2.5c_c \le 12\left(\frac{36}{f_s}\right)$$

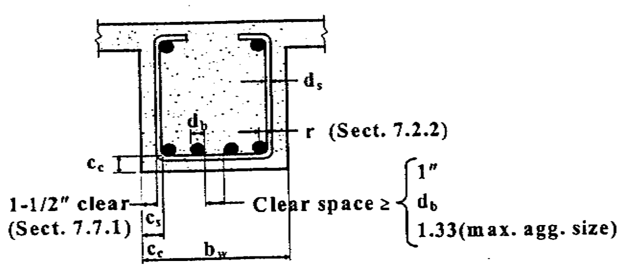


Figure 2. Cover and spacing requirements.

Table 1 contains the minimum number of bars required in a single layer for beams of various widths, assuming Grade 60 reinforcement, clear cover to the tension reinforcement c_c equal to 2 in., and f_s equal to 36 ksi.

The maximum number of bars n_{max} permitted in a section can be computed from (see Figure 2):

$$n_{max} = \frac{b_w - 2(c_s + d_s + r)}{(\text{Clear space}) + d_b} + 1$$

Table 2 contains the maximum number of bars permitted in a single layer assuming Grade 60 reinforcement, clear cover to the stirrups c_s equal to 1.5 in., and ¾-in. aggregate. Also, it is assumed that No. 3 stirrups are utilized for No. 5 and No. 6 longitudinal bars, and No. 4 stirrups are used for No. 7 and larger bars.

Selecting bars within the limits of Tables 1 and 2 will provide automatic conformance with the code requirements for cover and spacing, based on the assumptions noted above. Tables for other parameters can easily be generated.

The maximum bar spacing for one-way slabs is 12 in., assuming ¾-in. cover and f_s equal to 36 ksi.

Table 1. Minimum number of bars in a single layer per Sect. 10.6.

Bar	Beam width (in.)												
Size	12	14	16	18	20	22	24	26	28	30	36	42	48
No. 4	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 5	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 6	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 7	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 8	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 9	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 10	2	2	3	3	3	3	3	4	4	4	5	5	6
No. 11	2	2	3	3	3	3	3	4	4	4	5	5	6

Table 2. Maximum number of bars permitted in a single layer.

Bar	Beam width (in.)												
Size	12	14	16	18	20	22	24	26	28	30	36	42	48
No. 4	5	6	8	9	10	12	13	14	16	17	21	25	29
No. 5	5	6	7	8	10	11	12	13	15	16	19	23	27
No. 6	4	6	7	8	9	10	11	12	14	15	18	22	25
No. 7	4	5	6	7	8	9	10	11	12	13	17	20	23
No. 8	4	5	6	7	8	9	10	11	12	13	16	19	22
No. 9	3	4	5	6	7	8	8	9	10	11	14	17	19
No. 10	3	4	4	5	6	7	8	8	9	10	12	15	17
No. 11	3	3	4	5	5	6	7	8	8	9	11	13	15

When designing for the effects of shear, Equation (11-2) must be satisfied at any section:

$$V_u \le \phi V_n = \phi(V_c + V_s)$$

The selection and spacing of stirrups can be simplified if the spacing (s) is expressed as a function of the effective depth (d). Using three standard stirrup spacings (s = d/2, d/3, and d/4), values of ϕV_s can be derived for various stirrup sizes and spacings, independent of the member size.

For vertical stirrups (Equation 11-15):

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$

By substituting d/n for s (where n = 2, 3, or 4), the above equation can be rewritten as:

$$\phi V_s = \phi A_v f_y n$$

Thus, for Grade 60 No. 3 U-stirrups spaced at d/2:

$$\phi V_s = 0.85 \times 0.22 \times 60 \times 2 = 22.4 \text{ kips, say } 22 \text{ kips}$$

Table 3 contains values of ϕV_s for Grade 60 U-stirrups with 2 legs. It is important to reemphasize that values of ϕV_s are not dependent on the member size or on the concrete strength.

Table 3. Values of ϕV_s (f_y = 60 ksi).*

Stirrup spacing, s	No. 3 U-stirrup	No. 4 U-stirrup	No. 5 U-stirrup
d/2	22 kips	40 kips	63 kips
d/3	33 kips	61 kips	94 kips
d/4	44 kips	81 kips	126 kips

*Valid for stirrups with 2 legs (double the tabulated values for 4 legs, etc.)

Once $\phi V_s = V_u - \phi V_c$ has been computed at a particular section, a value of ϕV_s can be chosen from Table 3 that is equal to or slightly greater than that which is required. The stirrup size and spacing corresponding to this value of ϕV_s can be specified at this section of the beam.

For beams in special moment frames, V_u is computed in accordance with Section 21.3.4.1. For cases where V_c must be set equal to zero according to Section 21.3.4.2, hoop spacing can be determined from Table 3 based on V_u at the section. Section 21.3.3.2 gives maximum hoop spacing in regions where hoops are required. Outside of these regions, spacing of stirrups with seismic hooks can be determined utilizing V_c.

According to Section 11.5.6.9, ϕV_s is limited to $\phi 8\sqrt{f'_c} b_w d$. If it is determined that the required ϕV_s is greater than this limiting value, one or more of the cross-sectional dimensions must be increased in order to carry the factored shear force.

Larger stirrup sizes at wider spacings are more cost effective than smaller stirrup sizes at closer spacings, since the latter requires disproportionately high costs for fabrication and placement. Changing the stirrup spacing as few times as possible over the length of the member also results in cost savings.

In order to adequately develop the stirrups, the requirements of Section 12.13 must be satisfied. To allow for bend radii at corners of U-stirrups, the minimum beam widths given in Table 4 should be provided.

Table 4. Minimum beam widths.

Stirrup Size	Minimum Beam Width, b_w
No. 3	10 in.
No. 4	12 in.
No. 5	14 in.

In 1995, a new design procedure for torsion was introduced in ACI 318. The provisions are based on a thin-walled tube, space truss analogy, where the contribution of concrete to torsional strength is disregarded for simplicity. Except for a few minor changes, the torsion provisions in ACI 318-99 are virtually the same as those in ACI 318-95.

A comprehensive set of design aids that can be used to efficiently design and detail concrete beams subjected to the combined effects of flexure, shear, and torsion can be found in *Design of Concrete Beams for Torsion* (Fanella and Rabbat, 1997).

Reinforcement details for beams in intermediate and special moment frames can be found in *Seismic Detailing of Concrete Buildings* (Fanella, 2000).

Two-way Slabs

Calculation of deflections for two-way slabs is complicated, even when linear elastic behavior is assumed. To avoid extremely complex calculations in routine designs, deflections of non-prestressed two-way slab systems need not be computed if the slab thickness meets the minimum requirements of Section 9.5.3. Figure 3 contains minimum slab thickness as a function of clear span length ℓ_n in the long direction, assuming Grade 60 reinforcement and, if applicable, relatively stiff edge and/or interior beams.

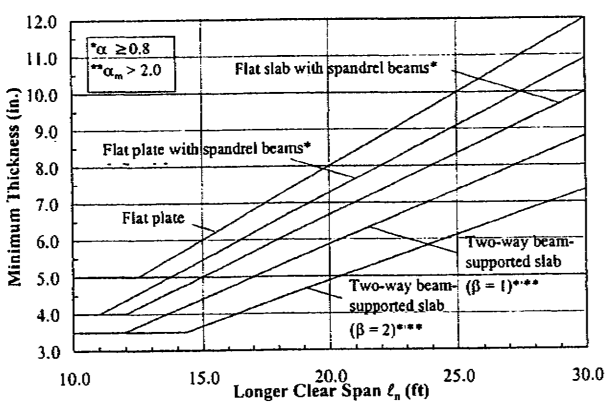


Figure 3. Preliminary slab thickness for two-way slab systems per Section 9.5.3.

When two-way slabs are supported directly on columns (as in flat plates and flat slabs), shear near the columns is of critical importance, especially at exterior slab-column connections without spandrel beams. For flat plates, the thickness of the slab will almost always be governed by two-way shear and not by serviceability requirements. Figure 4, which is based on the two-way shear requirements of Section 11.12, can be utilized to determine a preliminary slab thickness for a flat plate with square edge columns of size c_1 (3-sided critical section) supporting a tributary area (A), square bays ($\ell_1 = \ell_2$), gravity load moment transferred between the slab and edge column in accordance with the provisions of Section 13.6.3.6, and 4,000 psi, normal weight concrete. A preliminary slab thickness (h) can be obtained by adding 1.25 in. to d from the figure, where the total factored load w_u includes an estimate for the weight of the slab. For rectangular bays with $\ell_1 = 2\ell_2$, d from Figure 4 should be increased by about 15%, and for bays with $\ell_1 = 0.5\ell_2$, d may be decreased by 15%.

Fire resistance requirements per the governing building code must also be considered when specifying minimum slab thickness.

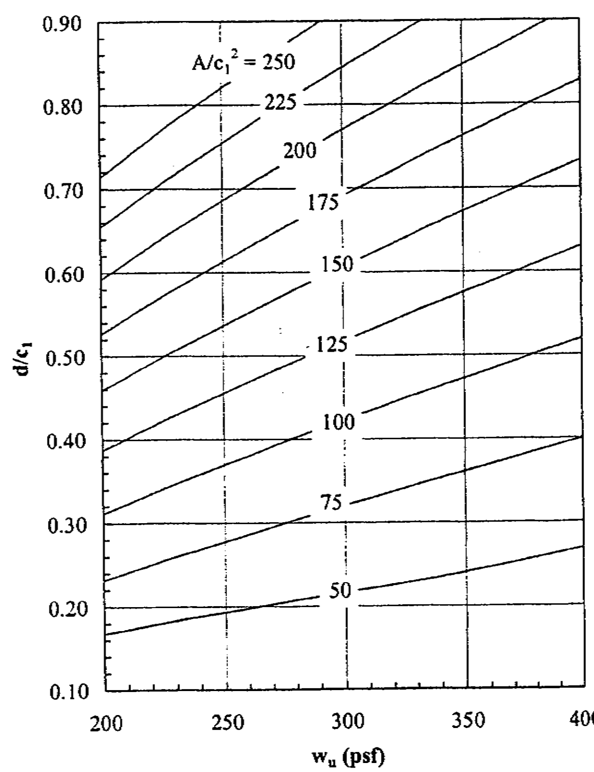


Figure 4. Preliminary slab thickness for flat plate construction based on two-way shear at edge column (f'_c = 4 ksi).

According to Section 13.5.1.1, either the Direct Design Method (DDM) of Section 13.6 or the Equivalent Frame Method (EFM) of Section 13.7 is permitted to determine the effects of gravity loads on two-way slab systems in lieu of other methods that satisfy conditions of equilibrium and geometric compatibility. For routine cases, the DDM can be used to quickly and easily determine moments in column and middle strips, as long as the conditions in Section 13.6.1 are satisfied. Moment redistribution as permitted by Section 8.4 shall not be applied for slab systems designed by the DDM.

The DDM is a three-step analysis procedure. The first step is calculation of the total factored static moment (M_o) for a panel. The second step involves distribution of M_o to negative and positive moment sections. In the third step, negative and positive factored moments are distributed to column and middle strips and to beams, if any.

For uniform loading, M_o for a panel is computed from the following (Equation (13-3)):

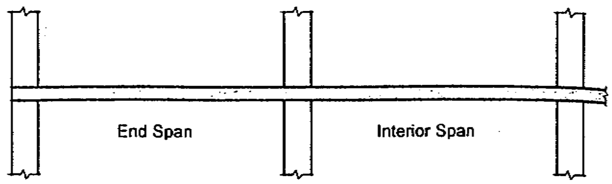
$$M_o = \frac{w_u \ell_2 \ell_n^2}{8}$$

where w_u is the factored load per unit area. Clear span ℓ_n is defined in Section 13.6.2.5 for both rectangular and nonrectangular supports, and span ℓ_2 is transverse to ℓ_n. Sections 13.6.2.3 and 13.6.2.4 provide definitions of ℓ_2 when the transverse span on either side of the centerline of supports varies and when the span adjacent and parallel to an edge is being considered, respectively.

Distribution of M_o into negative and positive moments, and then into column and middle strip moments (and, if applicable, beam moments), involves direct application of the moment coefficients contained in Sections 13.6.3 through 13.6.6. For design convenience, moment coefficients for flat plates/flat slabs and two-way beam-supported slabs are contained in Tables 5 and 6, respectively. Final moments in column and middle strips can be computed directly from the tabulated values, where all negative moments are at the face of the support.

Once the factored negative and positive moments (M_u) are determined from the applicable table, A_s can be readily computed using the equation developed above.

Table 5. Design moment coefficients used with the DDM for flat plates or flat slabs.



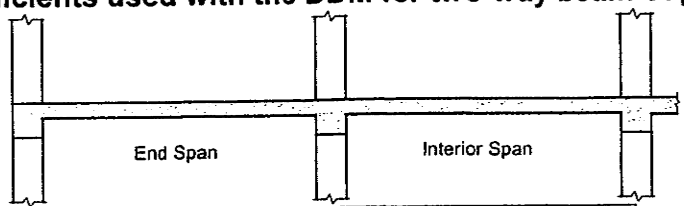
	Exterior negative	Positive	First interior negative	Positive	Interior negative
Total Moment	$0.26M_o$	$0.52M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.26M_o$	$0.31M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.21M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

According to Section 13.3, the minimum reinforcement ratio in each direction based on gross concrete area is 0.0018 for Grade 60 reinforcement, and maximum bar spacing is the smaller of 2h or 18 in.

Numerous analytical procedures exist for modeling frames subjected to lateral loads. In general, any procedure that satisfies equilibrium and geometric compatibility may be utilized, as long as results from the analysis are in reasonable agreement with test data. For slab-column frames, where only a portion of the slab is effective across its full width in resisting the effects of lateral loads, acceptable approaches include finite element models, effective beam width models, and equivalent frame models. Regardless of the method utilized, frame member stiffness must take into account effects of cracking and reinforcement so that drift due to wind and/or earthquake effects is not underestimated (Section 13.5.1.2).

Table 6. Design moment coefficients used with the DDM for two-way beam-supported slabs.



ℓ_2/ℓ_1	Moments		Exterior negative	Positive	First interior negative	Positive	Interior negative
	Total		$0.16M_o$	$0.57M_o$	$0.70M_o$	$0.3M_o$	$0.65M_o$
0.5	Column Strip	Beam	$0.12M_o$	$0.43M_o$	$0.54M_o$	$0.27M_o$	$0.50M_o$
		Slab	$0.02M_o$	$0.08M_o$	$0.09M_o$	$0.05M_o$	$0.09M_o$
	Middle Strip		$0.02M_o$	$0.06M_o$	$0.07M_o$	$0.03M_o$	$0.06M_o$
1.0	Column Strip	Beam	$0.10M_o$	$0.37M_o$	$0.45M_o$	$0.22M_o$	$0.42M_o$
		Slab	$0.02M_o$	$0.06M_o$	$0.08M_o$	$0.04M_o$	$0.07M_o$
	Middle Strip		$0.04M_o$	$0.14M_o$	$0.17M_o$	$0.09M_o$	$0.16M_o$
2.0	Column Strip	Beam	$0.06M_o$	$0.22M_o$	$0.27M_o$	$0.14M_o$	$0.25M_o$
		Slab	$0.01M_o$	$0.04M_o$	$0.05M_o$	$0.02M_o$	$0.04M_o$
	Middle Strip		$0.09M_o$	$0.31M_o$	$0.38M_o$	$0.19M_o$	$0.36M_o$

Notes: (1) All negative moments are at face of support.
(2) Beams and slabs satisfy stiffness criteria: $\alpha_1 \ell_2 / \ell_1 \geq 1.0$ and $\beta_t \geq 2.5$.

For flat plate frames, the effective beam width model will give reasonably accurate results in routine situations. In this method, the actual slab is replaced by a flexural element that has the same thickness as the slab and an effective beam width (b_e) that is a fraction of the actual transverse width of the slab. The following equation can be used to determine b_e for an interior slab-column frame (Hwang and Moehle, 2000):

$$b_e = 2c_1 + \frac{\ell_1}{3}$$

For an exterior frame, b_e equals one-half of the value computed from the above equation. It is shown in the reference that this solution produces an accurate estimate of elastic stiffness for regular frames.

To account for cracking in non-prestressed slabs, bending stiffness is typically reduced to between one-half and one-quarter of the uncracked stiffness, which is a function of h and b_e. When determining drifts or moment magnification in columns, lower-bound slab stiffness should be assumed. When slab-column frames interact with structural walls, a range of slab stiffnesses should be investigated in order to assess the importance of interaction.

According to Section 13.5.1.3, it is permitted to combine the results of the gravity load analysis with those of the lateral load analysis. Required reinforcement can be computed from the simplified equation above for the governing load combination.

When two-way slab systems are supported by beams or walls, shear forces in the slab are seldom a critical factor in design. In contrast, when two-way slabs are supported directly on columns, as in flat plates or flat slabs, shear around the columns is of critical importance, especially at exterior slab-column connections where the total exterior slab moment must be transferred directly to the column.

Two types of shear need to be investigated: one-way shear and two-way shear. For one-way or beam-type shear, which may be critical in long, narrow slabs, the critical section is at a distance d from the face of the support (Section 11.12.1.1). Design for one-way shear consists of checking that the following is satisfied at critical sections in both directions:

$$V_u \le \phi 2\sqrt{f'_c}\,\ell d$$

where ℓ is equal to ℓ_1 or ℓ_2 and V_u is the corresponding factored shear force at the critical section. Rarely is one-way shear a critical factor in design.

Two-way or punching shear is generally more critical than one-way shear in slab systems supported directly on columns. The critical section for two-way action is at a distance d/2 from edges or corners of columns, concentrated loads, reaction areas, and changes in slab thickness, such as edges of column capitals or drop panels (Section 11.12.1.2). For non-prestressed slabs of normal weight concrete without shear reinforcement, the following must be satisfied (Section 11.12.2):

$$v_u \le \text{smallest of } \phi v_c = \begin{cases} \phi\left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c} \\ \phi\left(2 + \dfrac{\alpha_s d}{b_o}\right)\sqrt{f'_c} \\ \phi 4\sqrt{f'_c} \end{cases}$$

where v_u is the maximum factored shear stress at the critical section and all other variables are defined in Section 11.0. Figure 5 contains allowable shear stress ϕv_c in slabs with 4,000 psi normal weight concrete at an edge column.

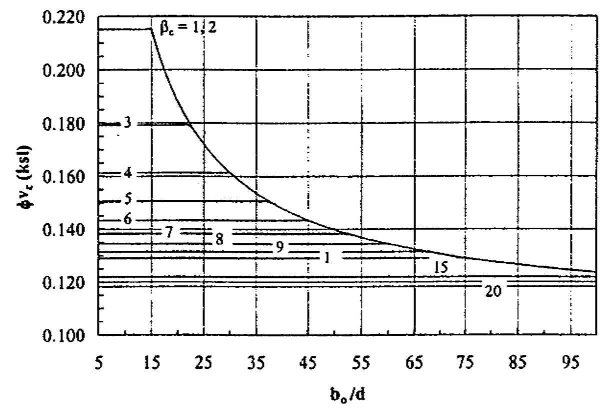


Figure 6. Two-way shear strength of slabs (f'_c = 4,000 psi) at an edge column (α_s = 30).

Transfer of moment in slab-column connections takes place by a combination of flexure (Section 13.5.3) and eccentricity of shear (Section 11.12.6). The portion of total unbalanced moment M_u transferred by flexure is $\gamma_f M_u$, where γ_f is defined in Equation (13-1). It is assumed that $\gamma_f M_u$ is transferred within an effective slab width equal to c_2 plus one and one-half slab or drop panel thicknesses on each side of the column or capital. Reinforcement is concentrated in the effective slab width such that $\phi M_n \ge \gamma_f M_u$ or, using the simplified equation, $A_s \ge \gamma_f M_u / 4d$. Under certain conditions, γ_f may be increased to values greater than those determined from Equation (13-1) (Section 13.5.3.3).

The portion of the total unbalanced moment M_u transferred by eccentricity of shear is $\gamma_v M_u$, where $\gamma_v = 1 - \gamma_f$ (Sections 13.5.3.1 and 11.12.6). When the DDM is used, the gravity load moment M_u to be transferred between slab and edge column must be $0.3M_o$ (Section 13.6.3.6).

Assuming that shear stress resulting from moment transfer by eccentricity of shear varies linearly about the centroid of the critical section, the factored shear stresses on the faces of the critical section are determined from the following (Section 11.12.6.2):

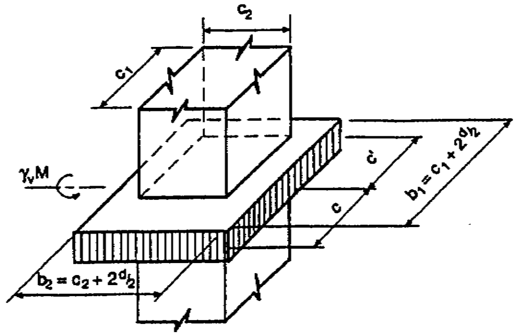
$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J/c}$$

$$v_{u2} = \frac{V_u}{A_c} - \frac{\gamma_v M_u}{J/c'}$$

where A_c is the area of the critical section and J/c and J/c′ are the section moduli of the critical section. As noted above, the maximum v_u must be less than or equal to the governing ϕv_c.

Numerous resources contain equations for determining A_c, J/c, and J/c (Fanella and Ghosh, 1993). Tables 7 through 9 facilitate calculation of these quantities for rectangular and circular columns.

Table 7. Properties of critical section–interior rectangular column.



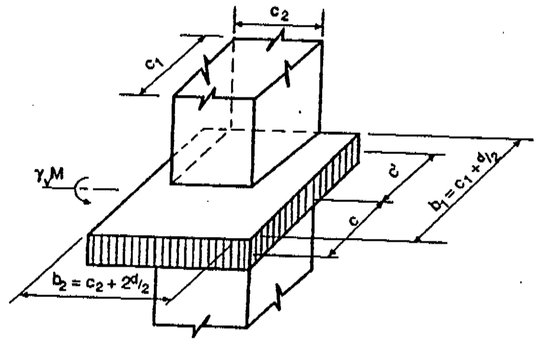
$c = c' = (c_1 + d)/2$

$A_c = f_1 d^2$

$J/c = J/c' = 2 f_2 d^3$

c_1/d	f_1							f_2						
	c_2/c_1							c_2/c_1						
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	7.00	7.50	8.00	8.50	9.00	9.50	10.00	2.33	2.58	2.83	3.08	3.33	3.58	3.83
1.50	8.50	9.25	10.00	10.75	11.50	12.25	13.00	3.40	3.86	4.33	4.80	5.27	5.74	6.21
2.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	4.67	5.42	6.17	6.92	7.67	8.42	9.17
2.50	11.50	12.75	14.00	15.25	16.50	17.75	19.00	6.15	7.24	8.33	9.43	10.52	11.61	12.71
3.00	13.00	14.50	16.00	17.50	19.00	20.50	22.00	7.83	9.33	10.83	12.33	13.83	15.33	16.83
3.50	14.50	16.25	18.00	19.75	21.50	23.25	25.00	9.73	11.70	13.67	15.64	17.60	19.57	21.54
4.00	16.00	18.00	20.00	22.00	24.00	26.00	28.00	11.83	14.33	16.83	19.33	21.83	24.33	26.83
4.50	17.50	19.75	22.00	24.25	26.50	28.75	31.00	14.15	17.24	20.33	23.43	26.52	29.61	32.71
5.00	19.00	21.50	24.00	26.50	29.00	31.50	34.00	16.67	20.42	24.17	27.92	31.67	35.42	39.17
5.50	20.50	23.25	26.00	28.75	31.50	34.25	37.00	19.40	23.86	28.33	32.80	37.27	41.74	46.21
6.00	22.00	25.00	28.00	31.00	34.00	37.00	40.00	22.33	27.58	32.83	38.08	43.33	48.58	53.83
6.50	23.50	26.75	30.00	33.25	36.50	39.75	43.00	25.48	31.57	37.67	43.76	49.85	55.95	62.04
7.00	25.00	28.50	32.00	35.50	39.00	42.50	46.00	28.83	35.83	42.83	49.83	56.83	63.83	70.83
7.50	26.50	30.25	34.00	37.75	41.50	45.25	49.00	32.40	40.36	48.33	56.30	64.27	72.24	80.21
8.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	36.17	45.17	54.17	63.17	72.17	81.17	90.17
8.50	29.50	33.75	38.00	42.25	46.50	50.75	55.00	40.15	50.24	60.33	70.43	80.52	90.61	100.71
9.00	31.00	35.50	40.00	44.50	49.00	53.50	58.00	44.33	55.58	66.83	78.08	89.33	100.58	111.83
9.50	32.50	37.25	42.00	46.75	51.50	56.25	61.00	48.73	61.20	73.67	86.14	98.60	111.07	123.54
10.00	34.00	39.00	44.00	49.00	54.00	59.00	64.00	53.33	67.08	80.83	94.58	108.33	122.08	135.83

Table 8. Properties of critical section–edge column, bending perpendicular to edge.



$$c = [f_3/(f_2 + f_3)][c_1 + d/2]$$

$$c' = [f_2/(f_2 + f_3)][c_1 + d/2]$$

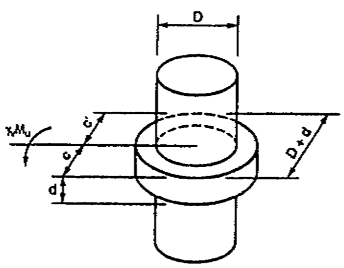
$$A_c = f_1 d^2$$

$$J/c = 2f_2 d^3$$

$$J/c' = 2f_3 d^3$$

c_1/d	f_1							f_2							f_3						
	c_2/c_1							c_2/c_1							c_2/c_1						
	0.50	0.75	1.00	1.25	1.50	1.75	2.00	0.50	0.75	1.00	1.25	1.50	1.75	2.00	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.0	4.50	4.75	5.00	5.25	5.50	5.75	6.00	1.38	1.51	1.65	1.79	1.93	2.07	2.21	0.69	0.70	0.71	0.72	0.72	0.73	0.74
1.5	5.75	6.13	6.50	6.88	7.25	7.63	8.00	2.07	2.34	2.60	2.87	3.14	3.40	3.67	1.11	1.13	1.16	1.18	1.19	1.21	1.22
2.0	7.00	7.50	8.00	8.50	9.00	9.50	10.00	2.94	3.38	3.81	4.24	4.68	5.11	5.54	1.63	1.69	1.73	1.77	1.80	1.82	1.85
2.5	8.25	8.87	9.50	10.13	10.75	11.38	12.00	3.98	4.62	5.26	5.91	6.55	7.19	7.83	2.27	2.36	2.43	2.49	2.53	2.58	2.61
3.0	9.50	10.25	11.00	11.75	12.50	13.25	14.00	5.18	6.08	6.97	7.86	8.76	9.65	10.54	3.02	3.15	3.25	3.34	3.41	3.46	3.51
3.5	10.75	11.62	12.50	13.37	14.25	15.12	16.00	6.56	7.74	8.93	10.11	11.30	12.48	13.67	3.89	4.06	4.20	4.31	4.41	4.49	4.56
4.0	12.00	13.00	14.00	15.00	16.00	17.00	18.00	8.10	9.62	11.13	12.65	14.17	15.69	17.21	4.86	5.09	5.27	5.42	5.55	5.65	5.74
4.5	13.25	14.37	15.50	16.62	17.75	18.87	20.00	9.80	11.70	13.59	15.49	17.38	19.27	21.17	5.94	6.24	6.47	6.66	6.82	6.95	7.06
5.0	14.50	15.75	17.00	18.25	19.50	20.75	22.00	11.68	13.99	16.30	18.61	20.92	23.23	25.54	7.14	7.51	7.80	8.03	8.22	8.38	8.51
5.5	15.75	17.12	18.50	19.87	21.25	22.62	24.00	13.72	16.49	19.26	22.03	24.80	27.56	30.33	8.44	8.89	9.24	9.52	9.76	9.95	10.11
6.0	17.00	18.50	20.00	21.50	23.00	24.50	26.00	15.93	19.20	22.46	25.73	29.00	32.27	35.54	9.86	10.40	10.82	11.15	11.43	11.65	11.85
6.5	18.25	19.87	21.50	23.12	24.75	26.37	28.00	18.30	22.11	25.92	29.73	33.54	37.36	41.17	11.39	12.02	12.51	12.91	13.23	13.50	13.72
7.0	19.50	21.25	23.00	24.75	26.50	28.25	30.00	20.84	25.24	29.63	34.02	38.42	42.81	47.21	13.03	13.77	14.34	14.79	15.17	15.47	15.74
7.5	20.75	22.62	24.50	26.37	28.25	30.12	32.00	23.55	28.57	33.59	38.61	43.63	48.65	53.67	14.78	15.63	16.29	16.81	17.24	17.59	17.89
8.0	22.00	24.00	26.00	28.00	30.00	32.00	34.00	26.42	32.11	37.80	43.48	49.17	54.86	60.54	16.64	17.61	18.36	18.95	19.44	19.84	20.18
8.5	23.25	25.37	27.50	29.62	31.75	33.87	36.00	29.47	35.86	42.25	48.65	55.04	61.44	67.83	18.61	19.71	20.56	21.23	21.78	22.23	22.61
9.0	24.50	26.75	29.00	31.25	33.50	35.75	38.00	32.67	39.82	46.96	54.11	61.25	68.40	75.54	20.69	21.93	22.88	23.63	24.25	24.75	25.18
9.5	25.75	28.12	30.50	32.87	35.25	37.62	40.00	36.05	43.98	51.92	59.86	67.79	75.73	83.67	22.89	24.27	25.33	26.17	26.85	27.41	27.89
10.0	27.00	29.50	32.00	34.50	37.00	39.50	42.00	39.59	48.36	57.13	65.90	74.67	83.44	92.21	25.19	26.72	27.90	28.83	29.59	30.21	30.74

Table 9. Properties of critical section– interior circular column.



$c = c' = (D + d)/2$

$A_c = f_1\, d^2$

$J/c = J/c' = 2f_2\, d^3$

D/d	f_1	f_2
1.00	6.28	1.74
1.50	7.85	2.62
2.00	9.42	3.70
2.50	11.00	4.98
3.00	12.57	6.45
3.50	14.14	8.12
4.00	15.71	9.98
4.50	17.28	12.05
5.00	18.85	14.30
5.50	20.42	16.76
6.00	21.99	19.41
6.50	23.56	22.26
7.00	25.13	25.30
7.50	26.70	28.54
8.00	28.27	31.98
8.50	29.85	35.61
9.00	31.42	39.44
9.50	32.99	43.46
10.00	34.56	47.68

Section 13.3 contains general reinforcement requirements for two-way slabs, including the minimum area of steel and maximum bar spacing. When choosing bar sizes, the largest bars that satisfy maximum limits on spacing will generally provide overall economy. Critical dimensions that limit bar size are thickness of slab available for hooks and distances from critical design sections to edges of slab.

Figure 13.3.8 contains minimum extensions for reinforcement in two-way systems without beams. These minimum lengths and extensions are usually not sufficient when a two-way slab is part of the lateral-force-resisting (LFR) system (Section 13.3.8.4). In such cases, bar lengths must be determined in accordance with Sections 12.10 through 12.12.

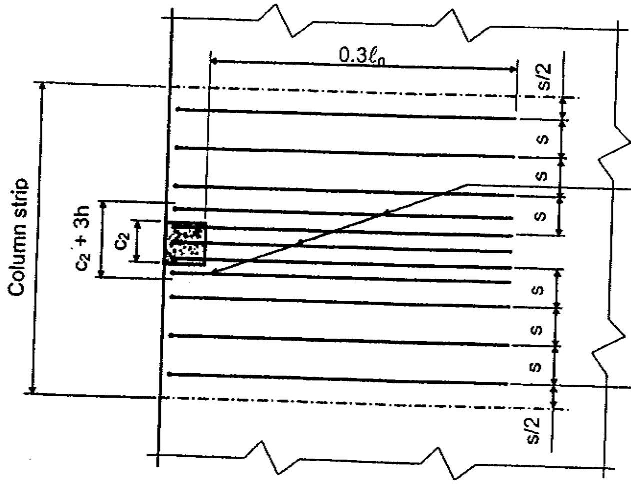
Details at edge and corner columns that satisfy the requirements of Section 13.5.3 for transfer of unbalanced moment by flexure are shown in Figure 7 for flat plates.

When two-way slab systems are part of the LFR system, distribution of moment transfer reinforcement at interior columns and at edge columns bending parallel to an edge depends on the ratio of the factored moments from gravity loads to factored moments from lateral loads in the slab. For ratios greater than 1, the combined moment in the slab on each face of the support is negative, and all of the moment transfer reinforcement should be placed at the top of the slab. However, for ratios less than 1, the combined moment is positive on one face of the support and negative on the other face. In this situation, it would be prudent to divide moment transfer reinforcement between the top and bottom of the slab, with the top and bottom reinforcement continuous over the column to account for moment reversals.

Columns

Total loads on columns are directly proportional to bay sizes (i.e., tributary areas). Therefore, larger bay sizes mean larger column sizes. Bay size is often dictated by architectural and functional requirements of a building. For example, larger bays are required to achieve maximum unobstructed floor space. The type of floor system that is utilized also affects column spacing: economical use of a non-prestressed flat plate floor system usually requires columns that are spaced closer than those supporting a one-way or two-way joist system.

Aside from the above considerations, it is important that columns satisfy all of the applicable strength requirements of ACI 318 and at the same time be economical. Since concrete is more cost effective than reinforcing steel for carrying axial compressive loads, it is typically more economical to use larger column sizes with lesser amounts of reinforcement.



Note: Where additional top bars are required, indicate the total number of bars on the design drawings as, for example, (8 + 3)-No. 4, where 8 is the number of uniformly spaced bars and 3 is the number of additional bars.

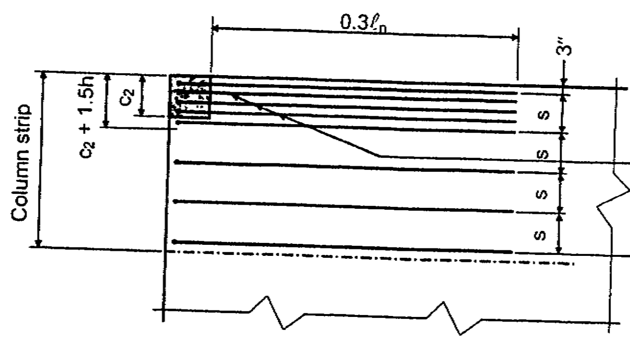


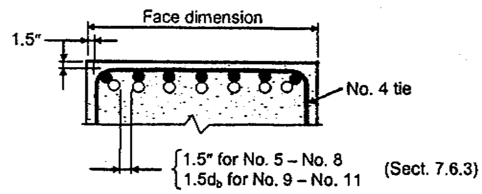
Figure 7. Typical reinforcement details at edge and corner columns in flat plates.

Columns must be sized not only for strength, but also for constructability. For proper concrete placement and consolidation, column and bar sizes should be selected to ensure that reinforcement is not congested, especially at beam-column joints. A smaller number of larger bars usually improves constructability. Table 10 can be used to determine the number of longitudinal bars that can be accommodated on the face of a rectangular tied column with normal lap splices, based on the provisions of Section 7.6.3. Similarly, Table 11 contains the maximum number of bars that can be accommodated in circular columns or square columns of size h with longitudinal reinforcement arranged in a circle with normal lap splices.

Reuse of column forms from story to story can result in significant savings. In low-rise buildings, it is more economical to use the same column size and concrete strength for the entire height of the building and to vary only the longitudinal reinforcement as required. In taller buildings, the size of the column can be changed over the height of the building, but the number of changes should be kept to a minimum. By judiciously varying reinforcement and concrete strength, the same column size can be used over a number of stories. In any size building, it is economically unsound to vary column size to suit the load on each story level.

In preliminary design, it is necessary to select column sizes for cost estimating and/or frame analysis. The initial selection can be very important when considering overall design time. A preliminary column size should be selected based on a low percentage of longitudinal reinforcement; in this way, additional reinforcement can be added in the final design stage without having to change the column size, which would typically require reanalysis of the structure. Columns with reinforcement ratios in the range of 1% to 2% are usually the most economical.

Table 10. Minimum face dimension (in.) of rectangular tied columns with normal lap splices



Bar Size	Number of bars per face												
	2	3	4	5	6	7	8	9	10	11	12	13	14
No. 5	8	10	12	14	17	19	21	23	25	27	29	31	34
No. 6	9	11	13	15	18	20	22	24	27	29	31	33	36
No. 7	9	11	14	16	18	21	23	26	28	30	33	35	37
No. 8	9	12	14	17	19	22	24	27	29	32	34	37	39
No. 9	10	13	16	18	21	24	27	30	33	35	38	41	44
No. 10	11	14	17	20	23	27	30	33	36	39	42	46	49
No. 11	11	15	18	22	25	29	32	36	40	43	47	50	54

*Column face dimension rounded to nearest inch.

Maximum size aggregate not larger than ¾ minimum clear spacing between bars (Section 3.3.2).

The following equations can be used to determine the minimum column face dimension for other cover and tie sizes:

- For No. 5 – No. 8:

  Minimum face dimension = $2(\text{cover} + \text{tie diameter}) + nd_b + 1.5(n - 1) + [(3 + 2d_b)\cos\theta - 0.586d_b - 3]$

- For No. 9 – No. 11:

  Minimum face dimension = $2(\text{cover} + \text{tie diameter}) + nd_b + 1.5d_b(n - 1) + 1.38d_b$

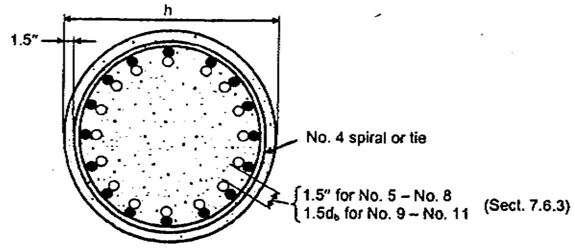
where n = number of bars per face

$$\theta = \arcsin\frac{0.2929d_b}{1.5 + d_b}$$

d_b = diameter of longitudinal bars

The design chart presented in Figure 8, based on Equation (10-2), can be used for nonslender tied columns loaded at an eccentricity of no more than about 10% of the overall thickness (i.e., columns with zero or small computed moments). Figure 8 provides quick estimates for gross area (A_g) of a column section required to support a factored axial load P_u within the reinforcement limits of Section 10.9, assuming Grade 60 reinforcement (f_y = 60 ksi). Appreciable bending moments can occur in columns due to unbalanced gravity loads and/or lateral forces. The *ACI Design Handbook* (ACI, 1997), the *CRSI Design Handbook* (CRSI, 2002) and the PCA computer program *PCACOL* (PCA, 1999) are a few of the many resources available to design columns for the combined effects of axial load and bending moment.

Table 11. Maximum number of bars in columns having bars arranged in a circle with normal lap splices*



h (in.)	Bar Size						
	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
12	8	7	6	6	4**	—	—
14	11	10	9	8	7	5**	4**
16	14	13	12	11	9	7	6
18	17	16	14	13	11	9	8
20	20	19	17	16	13	11	10
22	23	21	20	18	16	13	12
24	26	24	22	21	18	15	13
26	29	27	25	23	20	17	15
28	32	30	28	26	22	19	17
30	35	33	30	28	25	21	19
32	38	35	33	31	27	23	21
34	41	38	36	33	29	25	22
36	44	41	38	36	31	27	24
38	47	44	41	38	34	29	26
40	50	47	44	41	36	31	28
42	53	49	46	43	38	33	30
44	56	52	49	46	40	35	31
46	59	55	52	48	42	37	33
48	62	58	54	51	45	39	35

*Maximum size aggregate not larger than ¾ minimum clear spacing between bars (Section 3.3.2). Minimum number of longitudinal bars is 4 when enclosed by circular ties and 6 when enclosed by spirals (Section 10.9.2). Reinforcement ratios ρ_g are within limits of Section 10.9.1.

The following equation can be used to determine the maximum number of bars n for other cover and tie or spiral sizes:

$$n = \frac{180}{\arcsin\left[\dfrac{s/2}{0.5h - (\text{cover} + \text{tie or spiral diameter}) - 1.5d_b}\right]}$$

where $s = 1.5$ in. $+ d_b$ for No. 5 – No. 8

$= 1.5d_b + d_b = 2.5d_b$ for No. 9 – No. 11

d_b = diameter of longitudinal bars

**Applicable to circular tied columns only

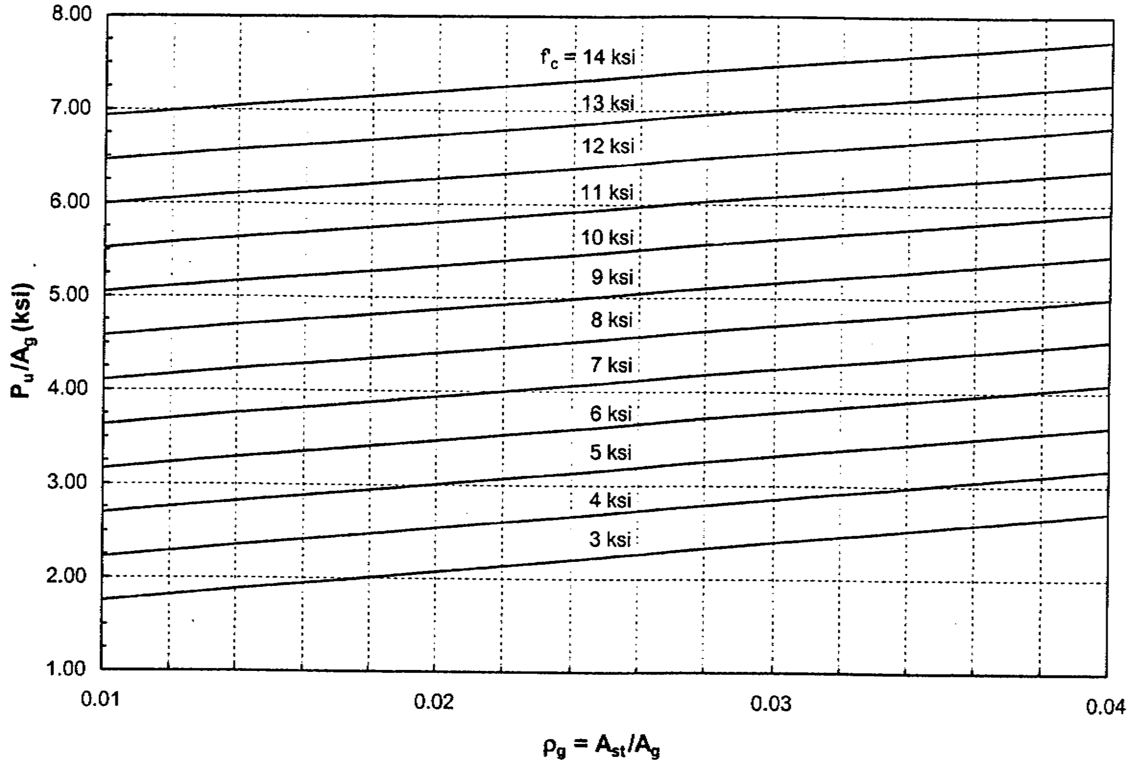


Figure 8. Design chart for nonslender, tied columns (f_y = 60 ksi).

A simplified interaction diagram, such as the one depicted in Figure 9, can be created by connecting straight lines between points corresponding to certain transition stages. The transition stages are:

- Stage 1: Pure compression (no bending)
- Stage 2: Stress in reinforcement closest to tension face = 0 ($f_s = 0$)
- Stage 3: Stress in reinforcement closest to tension face = $0.5f_y$ ($f_s = 0.5f_y$)
- Stage 4: Balanced point; stress in reinforcement closest to tension face = f_y ($f_s = f_y$)
- Stage 5: Pure bending (no axial load)

The type of lap splice that is required depends on where the load combinations fall within the interaction diagram (Section 12.17). For all load combinations falling within Zone 1, compression lap splices are allowed. In Zone 2, either Class A or Class B tension lap must be used, depending on the number of bars spliced at a section. Class B tension lap splices are required when one or more load combinations fall within Zone 3.

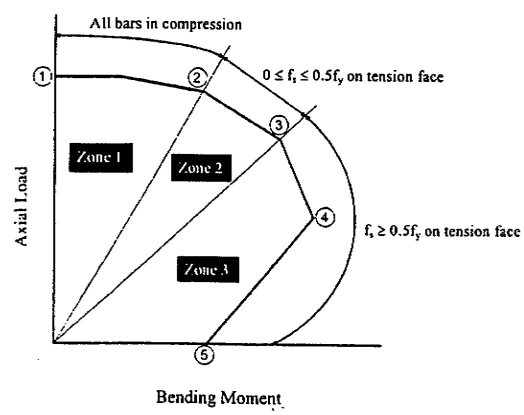


Figure 9. Transition stages on interaction diagram.

The information in Figure 10 can be utilized to determine the critical points on the interaction diagram for a rectangular tied column with Grade 60 bars arranged symmetrically in the section. The equations for ϕP_n and ϕM_n for points 2 through 4 yield larger values than the exact ones, since, for simplicity, the stress $0.85f'_c$ is not subtracted from the stress f_{si} in the bars located in the compression block. For columns with reinforcement ratios up to 3%, the difference between the computed and exact values of ϕP_n is at most 10%, with the difference much less than that for ϕM_n.

The reciprocal load method, which is described in Sections R10.3.5 and R10.3.6, is one of the simplified methods that can be utilized to design columns subjected to bending about both principal axes simultaneously. A trial section can be obtained from an interaction diagram using the factored axial load P_u and the total factored moment $M_u = M_{ux} + M_{uy}$, where M_{ux} and M_{uy} are the factored moments about the principal axes. The equation in Sections R10.3.5 and R10.3.6 can then be used to check if the section is adequate or not. Although this design process is conservative, only an adjustment in the amount of reinforcement will usually be necessary to obtain an adequate or more economical solution.

Provisions for slenderness effects are contained in Sections 10.10–10.13. According to Section 10.10, columns must be designed based on a second-order analysis or, when applicable, the magnified moment method of Sections 10.11–10.13. In the latter case, 2 methods are provided to determine whether stories in a structure are nonsway or sway (Section 10.11.3).

The maximum unsupported column length ℓ_u that would permit slenderness to be neglected for a nonsway (k = 1) rectangular column of size h that is bent in double curvature with equal factored end moments is (Section 10.12.2):

$$\frac{k\ell_u}{r} \leq 34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12(-1) = 46 > 40, \text{ use } 40$$

$$\frac{k\ell_u}{0.3h} \leq 40 \text{ or } \ell_u \leq 12h$$

Beam stiffnesses at the top and bottom of a column in a sway frame can have a significant influence on the degree of slenderness. For example, for a sway column with a column-to-beam stiffness ratio $\Psi = 1.0$ at both ends, the effective length factor $k = 1.3$ (see Figure R10.12.1), and the effects of slenderness may be neglected when $\ell_u < 5h$. If the beam stiffness is reduced to one-fifth of the column stiffness at each end, then $k = 2.2$, and slenderness effects need not be considered when $\ell_u < 3h$.

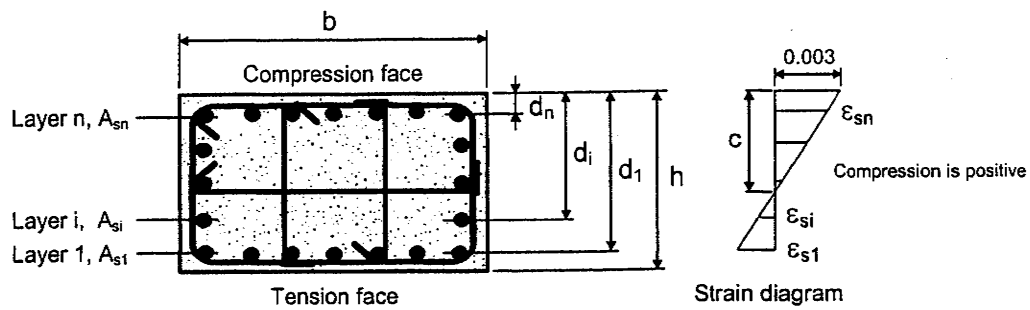
When slenderness effects must be considered, moments at column ends are to be magnified according to Sections 10.12 and 10.13 for nonsway and sway frames, respectively, or a second-order analysis must be performed. For columns subjected to biaxial bending, the moment about each axis shall be magnified separately. Columns are designed for combined effects of axial load and magnified bending moments utilizing the methods described above.

Details for column reinforcement can be found in *ACI Detailing Manual* (ACI, 1994), *Reinforcing Bar Detailing Manual* (CRSI, 2000), and *Simplified Design of Concrete Buildings of Moderate Size and Height* (Fanella and Ghosh, 1993). Details for columns in regions of high and moderate seismic risk are also available (Fanella, 2000).

Walls

For buildings in the low to moderate height range, frame action alone is usually sufficient to provide adequate resistance to lateral loads. Since frame buildings depend primarily on the rigidity of the slab-column or beam-column joints, they tend to be uneconomical beyond 11–14 stories in regions of high to moderate seismicity and 15–20 stories elsewhere. High-rise frames often could not be efficiently designed to satisfy strength and drift requirements without structural walls.

If possible, walls should be located within the plan of the building to minimize torsional effects from lateral loads. Since concrete floor systems act as rigid horizontal diaphragms, lateral loads are distributed to lateral-force-resisting elements in proportion to their rigidities.



Refer to Figure 9 for location of points on interaction diagram.

- Point 1: Pure compression (no bending)

$$\phi P_{n(max)} = 0.80\phi A_g[0.85f'_c + \rho_g(f_y - 0.85f'_c)] \quad \text{ACI Equation (10-2)}$$

- Points 2 – 4

$$\phi P_n = \phi\left[C_1 d_1 b + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\right] \quad \text{(kips)}$$

$$\phi M_n = \phi\left[0.5C_1 d_1 b\left(h - \frac{\beta_1 d_1}{C_2}\right) + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\left(\frac{h}{2} - d_i\right)\right]/12 \quad \text{(ft-kips)}$$

where $C_1 = 0.85f'_c\beta_1\left(\frac{0.003}{0.003 - \varepsilon_{s1}}\right)$

$$C_2 = \frac{0.003 - \varepsilon_{s1}}{0.003}$$

$\left(1 - C_2\frac{d_i}{d_1}\right) \le \frac{60}{87} = 0.69$ to ensure that the stress in the bars $\le f_y = 60$ ksi

$0.85 \ge \beta = 1.05 - 0.05f'_c \ge 0.65$ (Section 10.2.7.3; f'_c in ksi)

			f'_c (ksi)													
			4		5		6		7		8		9		10	
Point	f_{s1}/f_y	ε_{s1}	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
2	0.00	0.00000	2.89	1.00	3.40	1.00	3.83	1.00	4.17	1.00	4.42	1.00	4.97	1.00	5.53	1.00
3	-0.50	-0.00103	2.15	1.34	2.53	1.34	2.84	1.34	3.10	1.34	3.29	1.34	3.70	1.34	4.11	1.34
4	-1.00	-0.00207	1.71	1.69	2.01	1.69	2.26	1.69	2.47	1.69	2.62	1.69	2.94	1.69	3.27	1.69

- Point 5: Pure bending (no axial load)
  Use iterative procedure to determine ϕM_n.

Figure 10. Simplified interaction diagram for rectangular, tied columns with symmetrical reinforcement (f_y = 60 ksi).

Since walls are typically located around elevators and stairs, their lengths are usually dictated by the size of these openings. From a practical standpoint, a minimum thickness of 6 in. is required for a wall with a single layer of reinforcement and 10 in. for a wall with two layers.

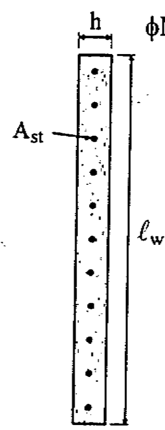
When walls are present in buildings of low to moderate height, frame-wall interaction can usually be neglected, since walls are normally stiff enough to attract the majority of the effects from lateral loads. This greatly reduces overall analysis and design time, and generally results in a nonsway frame. In contrast, frame-wall interaction must be considered for high-rise structures where the walls have a significant effect on the frame: in the upper stories, the frame must resist more than 100% of the story shear caused by lateral loads. Neglecting frame-wall interaction would not be conservative at these levels. Also, a more economical solution will be obtained when frame-wall interaction is considered.

According to Section 14.2.2, walls shall be designed in accordance with Section 14.2, 14.3, and either 14.4, 14.5, or 14.8. Section 14.4 contains the requirements for walls designed as compression members using the strength design provisions of Chapter 10, which are described above for columns. Any wall may be designed by this method, and no minimum wall thicknesses are prescribed. The Empirical Design Method of Section 14.5 and the Alternate Design Method of Section 14.8 may only be used when various conditions are satisfied.

In addition to the provisions of Chapter 14, walls in buildings located in regions of high seismic risk must be designed and detailed in accordance with Section 21.6. In certain situations, special transverse reinforcement is required at wall ends (Section 21.6.6). No special detailing is required in regions of moderate or low seismic risk.

In high-rise buildings, walls are ordinarily designed in accordance with Section 14.4 or, if applicable, Section 21.6. Interaction diagrams can be obtained utilizing the resources above for columns.

For buildings of low to moderate height, walls with uniform cross-sections and uniformly distributed vertical and horizontal reinforcement are usually the most economical. For rectangular walls containing uniformly distributed vertical reinforcement and subjected to an axial load (P_u) smaller than that producing balanced failure, the information in Figure 11 can be used to determine the nominal moment capacity of the wall. This method should apply in a majority of cases, since the axial loads are usually small.



$$\phi M_n = \phi\left[0.5 A_{st} f_y \ell_w \left(1 + \frac{P_u}{A_{st} f_y}\right)\left(1 - \frac{c}{\ell_w}\right)\right]$$

$$\frac{c}{\ell_w} = \frac{\omega + \alpha}{2\omega + 0.85\beta_1}$$

$$\omega = \left(\frac{A_{st}}{\ell_w h}\right)\frac{f_y}{f'_c}$$

$$\alpha = \frac{P_u}{\ell_w h f'_c}$$

Figure 11. Approximate nominal moment capacity of low-rise walls.

In regions of low and moderate seismic risk, shear provisions for walls are contained in Section 11.10. The amounts of vertical and horizontal reinforcement required for shear depend on the magnitude of the factored shear force (V_u):

- $V_u \leq \phi V_c/2$: Provide minimum reinforcement in accordance with Section 11.10.9 or Chapter 14
- $\phi V_c/2 < V_u \leq \phi V_c$: Provide minimum reinforcement in accordance with Section 11.10.9
- $V_u > \phi V_c$: Provide horizontal reinforcement in accordance with Equation (11-33)

where ϕV_c is defined in Section 11.10.5. Tables 12 and 13 contain minimum reinforcement per Chapter 14 and Section 11.10.9, respectively.

In regions of high seismic risk, the shear provisions of Section 21.6.4 must be satisfied.

Table 11. Minimum wall reinforcement ($V_u \le \phi V_c/2$).

Wall Thickness h (in.)	Vertical		Horizontal	
	Min. A_s* (in.²/ft)	Suggested Reinforcement	Min. A_s** (in.²/ft)	Suggested Reinforcement
6	0.09	No. 3 @ 15″	0.14	No. 4 @ 16″
8	0.12	No. 3 @ 11″	0.19	No. 4 @ 12″
10	0.14	No. 4 @ 16″	0.24	No. 5 @ 15″
12	0.17	No. 3 @ 15″	0.29	No. 4 @ 16″
14	0.20	No. 3 @ 13″	0.34	No. 4 @ 14″
16	0.23	No. 3 @ 11″	0.38	No. 4 @ 12″
18	0.26	No. 4 @ 18″	0.43	No. 5 @ 17″

*Min. A_s/ft = 0.0012(12)h = 0.0144h for Grade 60, No. 5 bars and smaller (Section 14.3.2)
**Min. A_s/ft = 0.0020(12)h = 0.0240h for Grade 60, No. 5 bars and smaller (Section 14.3.3)
Note: two layers of reinforcement are required for walls thicker than 10 in. (Section 14.3.4).

Table 12. Minimum wall reinforcement ($\phi V_c/2 < V_u \le \phi V_c$).

Wall Thickness h (in.)	Vertical and Horizontal	
	Min. A_s* (in.² /ft)	Suggested Reinforcement
6	0.18	No. 4 @ 13″
8	0.24	No. 5 @ 15″
10	0.30	No. 5 @ 12″
12	0.36	No. 4 @ 13″
14	0.42	No. 4 @ 11″
16	0.48	No. 5 @ 15″
18	0.54	No. 5 @ 13″

*Min. A_s/ft = 0.0025(12)h = 0.03h (Section 11.10.9)
Note: two layers of reinforcement are required for walls thicker than 10 in. (Section 14.3.4).

Design for required horizontal reinforcement in walls where $V_u > \phi V_c$ can be simplified by determining values of the design shear strength (ϕV_s) provided by the horizontal reinforcement. According to Equation (11-33):

$$\phi V_s = \frac{\phi A_v f_y d}{s_2}$$

where A_v is the area of horizontal shear reinforcement within a distance s_2 and $d = 0.8\ell_w$ (Section 11.10.4). For example, for a wall reinforced with a single layer of Grade 60 No. 4 bars spaced at 12 in.:

$$\phi V_s = \frac{0.85 \times 0.20 \times 60 \times (0.8 \times 12\ell_w)}{12} = 8.2\ell_w \text{ kips}$$

Table 13 contains values of ϕV_s per foot length of wall for various horizontal bar sizes and spacing.

Table 13. Shear strength ϕV_s provided by horizontal shear reinforcement (f_y = 60 ksi).*

Bar spacing s_2 (in.)	ϕV_s (kips/ft length of wall)			
	No. 3	No. 4	No. 5	No. 6
6	9.0	16.3	25.3	35.9
7	7.7	14.0	21.7	30.8
8	6.7	12.2	19.0	26.9
9	6.0	10.9	16.9	23.9
10	5.4	9.8	15.2	21.5
11	4.9	8.9	13.8	19.6
12	4.5	8.2	12.7	18.0
13	4.1	7.5	11.7	16.6
14	3.9	7.0	10.8	15.4
15	3.6	6.5	10.1	14.4
16	3.4	6.1	9.5	13.5
17	3.2	5.8	8.9	12.7
18	3.0	5.4	8.4	12.0

*Values of ϕV_s are for walls with a single layer of reinforcement. Tabulated values can be doubled for walls with two layers.

Once the required $\phi V_s = V_u - \phi V_c$ is computed, a bar size and spacing that provides at least that amount of shear strength can easily be chosen from Table 13.

Required vertical shear reinforcement is determined from Equation (11-34) in regions of low and moderate seismic risk. When the wall height-to-length ratio $h_w/\ell_w < 0.5$, the amount of vertical reinforcement is equal to the amount of horizontal reinforcement. When $h_w/\ell_w > 2.5$, the minimum amount of vertical reinforcement in Table 12 is required.

Maximum spacing of horizontal and vertical bars is given in Sections 11.10.9.3 and 11.10.9.5, respectively.

Conclusion

The design aids presented in this paper, which are based on the provisions of ACI 318-99, can be used to significantly decrease design and detailing time required for beams, one-way slabs, two-way slabs, columns, and walls. The PCA Web site contains design examples illustrating the use of the timesaving design methods presented here (www.portcement.org/buildings).

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