Mohr's Circle Rules and Comments
July 1, 2012

2D Mohr's Circle

1. Mohr's circle is plotted on a cartesian coordinate system as shown in Figure 1, where positive normal stress coordinate, σ, is to the right and positive shear stress coordinate, τ, is downwards.

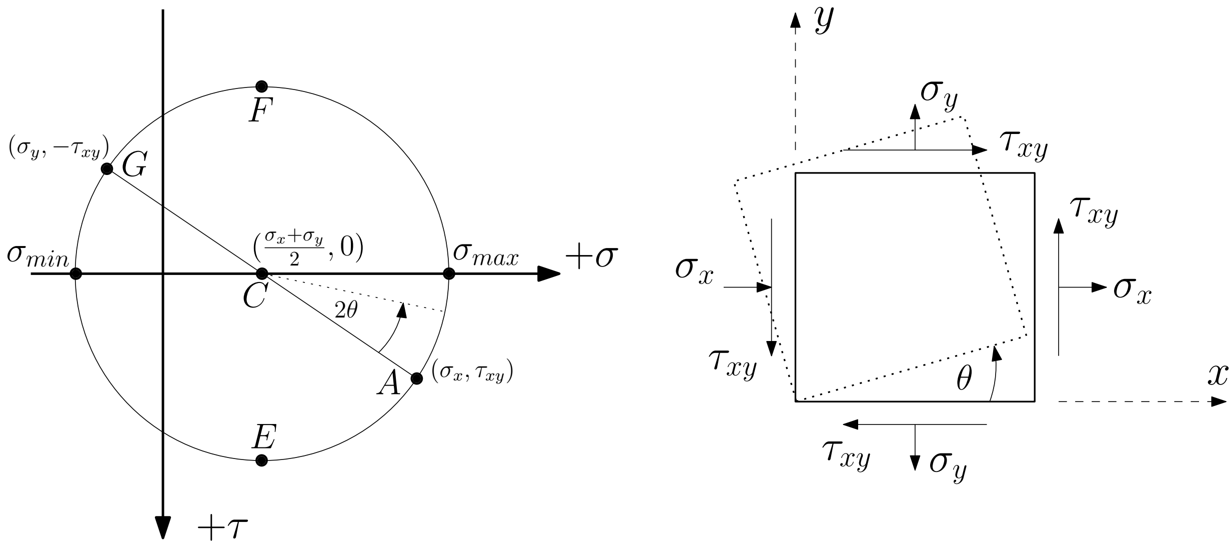

Figure 1: Positive coordinate system for Mohr's circle.

Shear stress is measured downwards so that rotations on Mohr's circle occur in the same direction as rotations on the stress block currently being analyzed.

2. The center of the circle, C, is located at coordinate $(\frac{\sigma_x+\sigma_y}{2}, 0)$.

3. A reference point, A, is created for the given stress block at coordinate (σ_x, τ_{xy}).

4. Points C and A define the radius of the Mohr's circle for the given stress block.

5. From the center and having the radius, Mohr's circle may be drawn.

6. The principle stresses will lie on the σ axis. The principle stresses are often referred to as $\sigma_{max} = \sigma_1$ and $\sigma_{min} = \sigma_2$.

7. The maximum in plane shear stress is located at the top, point F (or bottom, point E) of the circle and is located above (or below) the σ_{avg} coordinate.

8. For the given stress block, after Mohr's circle has been constructed, the state of stress at any angle, θ, from our given stress block may be found. This may be accomplished by moving 2θ away from radius, CA, on Mohr's circle and reading off the resulting (σ_x, τ_{xy}) values. The stress σ_y is 90 degrees away on the stress block, so it is 180 degrees away on Mohr's circle and can be read from the circle also (see point G). Positive angles are always measured counterclockwise usually from the positive x-axis of the given stress block, or from radius CA on Mohr's circle.

3D Mohr's Circle

Using the equalities $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{zy}$ the state of stress at any point may be written in matrix form as follows:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix} \tag{1}$$

The eigenvalues of the stress matrix are the principle stresses. That is, the principle stresses are the roots of the characteristic equation found by taking the determinant of $\boldsymbol{\tau} - \sigma \boldsymbol{I}$, where σ can be any one of the three eigenvalues (or principle stresses). After some algebra the characteristic equation is

$$\sigma^3 - (\tau_{xx} + \tau_{yy} + \tau_{zz})\sigma^2 + (\tau_{xx}\tau_{yy} + \tau_{yy}\tau_{zz} + \tau_{zz}\tau_{xx} - \tau_{xz}^2 - \tau_{xy}^2 - \tau_{yz}^2)\sigma - (\tau_{xx}\tau_{yy}\tau_{zz} - \tau_{xx}\tau_{yz}^2 - \tau_{yy}\tau_{xz}^2 - \tau_{zz}\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{xz}) = 0 \tag{2}$$

The above equation may be rewritten as

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0, \tag{3}$$

where I_1, I_2 and I_3 are the stress invariants defined as

$$I_1 = \tau_{xx} + \tau_{yy} + \tau_{zz} = tr\boldsymbol{\tau} \tag{4a}$$

$$I_2 = \tau_{xx}\tau_{yy} + \tau_{yy}\tau_{zz} + \tau_{zz}\tau_{xx} - \tau_{xz}^2 - \tau_{xy}^2 - \tau_{yz}^2 = \frac{1}{2}\left[(tr\boldsymbol{\tau})^2 - tr(\boldsymbol{\tau}^2)\right] \tag{4b}$$

$$I_3 = \tau_{xx}\tau_{yy}\tau_{zz} - \tau_{xx}\tau_{yz}^2 - \tau_{yy}\tau_{xz}^2 - \tau_{zz}\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{xz} = det\boldsymbol{\tau}. \tag{4c}$$

In two dimensions the above characteristic equation (2) simplifies to

$$\sigma^2 - (\tau_{xx} + \tau_{yy})\sigma + (\tau_{xx}\tau_{yy} - \tau_{xy}^2) = 0 \tag{5}$$

since $\tau_{zz} = \tau_{xz} = \tau_{yz} = 0$.

So for a given stress block with given stresses τ_{ij} the principle stresses σ_1, σ_2 and σ_3 may be found. From the largest and smallest of these three stresses the maximum shear stress is found as

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{6}$$

Suppose for example the principle stresses were ($\sigma_1 = 50$ ksi, $\sigma_2 = 10$ ksi, $\sigma_3 = -15$ ksi). Then 3D Mohr's circle can be plotted as shown below in Figure 2.

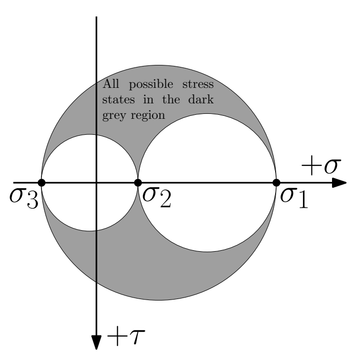

Figure 2: 3D Mohr's Circle Example