Meshfree Inelastic Frame Analysis

Theory & Results

Louie L. Yaw, Sashi Kunnath and N. Sukumar

University of California, Davis
Department of Civil and Environmental Engineering

Minisymposium 47 – Recent Advances in Modeling of Engineering Materials/Systems
USNCCM9 – San Francisco
July 24, 2007

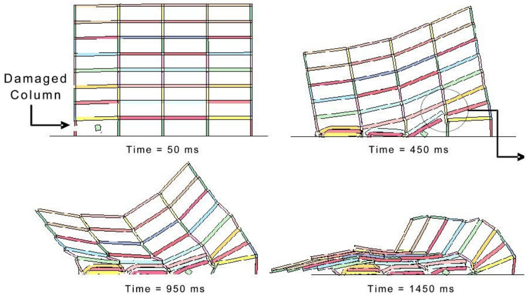
Acknowledgements

- Funding provided by Walla Walla College
- N. Sukumar acknowledges research support of NSF (Grant CMMI-0626481)
- Helpful discussions with Dr. Michael Puso, Lawrence Livermore National Laboratory
- Helpful discussions with Professor Boris Jeremic, UC Davis

Outline

Motivation

- Goal to advance collapse simulation technology
- Current FE technology unsatisfactory for large deformations at collapse limit states
- To explore feasibility of meshfree approach



Outline

MLS Shape Functions Derivation

- Start with displacement approximation

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_{a=1}^{n} \phi_a(\boldsymbol{x})\boldsymbol{d}_a \equiv \boldsymbol{\phi}^T \boldsymbol{d}$$

- Shape function ϕ_a is of the form (Belytschko et al 1996)

$$\phi_a(\boldsymbol{x}) = \boldsymbol{P}^T(\boldsymbol{x}_a)\alpha(\boldsymbol{x})w(\boldsymbol{x}_a),$$

where $\boldsymbol{P}(\boldsymbol{x}) = \{1\ x\ y\}^T$ is a linear basis in two dimensions, $\alpha(\boldsymbol{x})$ is a vector of unknowns to be determined and $w(\boldsymbol{x}) \geq 0$ is a weighting function

MLS Shape Functions Derivation

- The ϕ's must satisfy *reproducing conditions*

$$\boldsymbol{P}(\boldsymbol{x}) = \sum_{a=1}^{n} \boldsymbol{P}(\boldsymbol{x}_a)\phi_a(\boldsymbol{x})$$

- Substitution of ϕ_a into $\boldsymbol{P}(\boldsymbol{x})$ and solving for α yields

$$\alpha(\boldsymbol{x}) = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})$$

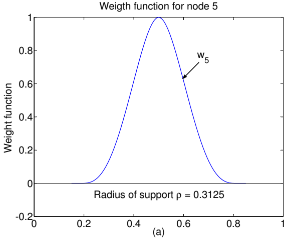
- Finally, substituting α into ϕ_a gives

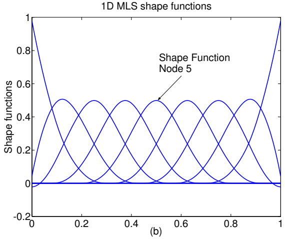
$$\boxed{\phi_a(\boldsymbol{x}) = \boldsymbol{P}^T(\boldsymbol{x}_a)\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})w(\boldsymbol{x}_a)},$$

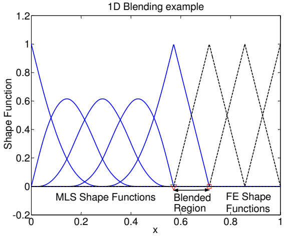
where $\boldsymbol{A} = \sum_{a=1}^{n} \boldsymbol{P}(\boldsymbol{x}_a)\boldsymbol{P}^T(\boldsymbol{x}_a)w(\boldsymbol{x}_a)$

Enforcing boundary conditions

- MLS (meshfree) shape functions do not have the Kronecker-delta property
- Hence MLS shape functions are blended with quadrilateral FE shape functions at essential B.C.'s
- Then essential boundary conditions are enforced on the finite element nodes in the standard way
- The blending technique proposed by Huerta and Fernández-Méndez (2004) is adopted


Weigth function for node 5
Weight function
w_5
Radius of support ρ = 0.3125
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
(a)


1D MLS shape functions
Shape functions
Shape Function
Node 5
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
(b)


1D Blending example
Shape Function
MLS Shape Functions
Blended
Region
FE Shape
Functions
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
x

Nodal Integration

- *Smoothed* strain tensor for node a (per Chen et al (2001))

$$\varepsilon_{ij}(\boldsymbol{x}_a) = \frac{1}{2A_a}\int_{V_a}(u_{i,j} + u_{j,i})\, dV = \frac{1}{2A_a}\int_{S_a}(u_i n_j + u_j n_i)\, dS$$

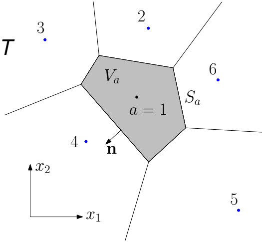
- Strain-displacement relation

$$\varepsilon(\boldsymbol{x}_a) = \sum_{b=1}^{6} \boldsymbol{B}_b(\boldsymbol{x}_a)\boldsymbol{d}_b \equiv \mathbf{Bd}$$

Nodal Integration

- Strain-displacement definitions

$$\boldsymbol{\varepsilon} = [\varepsilon_{11}\ \varepsilon_{22}\ 2\varepsilon_{12}]^T \quad \text{and} \quad \boldsymbol{d}_a = [d_{a1}\ d_{a2}]^T$$



$$\boldsymbol{B}_b(\boldsymbol{x}_a) = \begin{bmatrix} b_{b1}(\boldsymbol{x}_a) & 0 \\ 0 & b_{b2}(\boldsymbol{x}_a) \\ b_{b2}(\boldsymbol{x}_a) & b_{b1}(\boldsymbol{x}_a) \end{bmatrix}$$

$$b_{bi}(\boldsymbol{x}_a) = \frac{1}{A_a} \int_{S_a} \phi_b(\boldsymbol{x}) n_i(\boldsymbol{x})\, dS$$

- Nodally integrated stiffness matrix

$$\boldsymbol{K}_{bc} = \sum_{a=1}^{n} \boldsymbol{B}_b^T(\boldsymbol{x}_a)\boldsymbol{C}\boldsymbol{B}_c(\boldsymbol{x}_a)A_a t$$

Outline

Stabilization of Nodal Integration

- Nodal integration w/o stabilization leads to
  a. hourglass modes
  b. spurious low energy modes
  c. and locking
- Following Puso and Solberg (2006) stabilization is provided to the stiffness matrix as follows:

$$\boldsymbol{K}^{s} = (1 - \alpha_s)\boldsymbol{K}^{MLS} + \alpha_s \boldsymbol{K}^{FE},$$

where $\boldsymbol{K}^{s}$ is the stabilized matrix and $\alpha_s = 0.05$ is called the stabilization factor

Nonlinear analysis

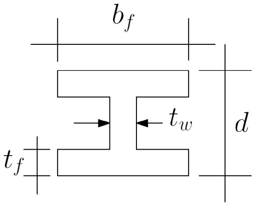
- Loads applied incrementally
- At global level a Newton-Raphson scheme is used to iterate the linearized system of equations until equilibrium is achieved

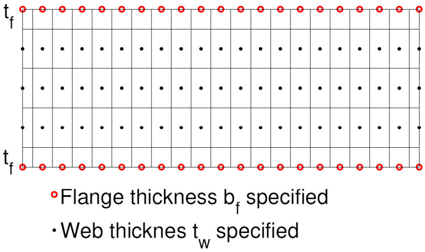
$$\boldsymbol{K}_{n+1}^{t(\nu)} \Delta \boldsymbol{d}_{n}^{(\nu)} = \boldsymbol{f}_{n+1}^{ext} - \boldsymbol{f}_{n+1}^{int(\nu)}$$

- At the constitutive level for $J2$ plasticity a radial return scheme is used (Simo and Hughes (1998))

Meshfree analysis of wide-flange steel sections

- Based on the meshfree nodal discretization of a beam a Voronoi diagram is generated.
- A thickness is specified for each Voronoi cell.
- To get wide-flange behavior a web thickness and a flange thickness is specified.





Outline

Linear elastic cantilever I-beam

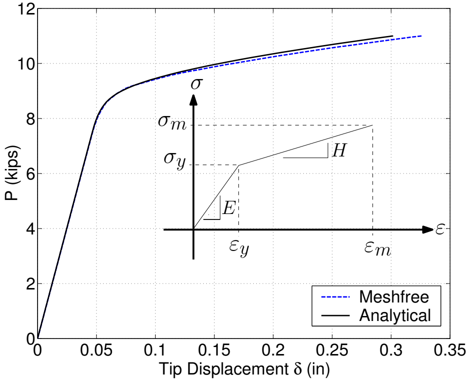
Results for normalized tip displacement and maximum bending stress, where $\delta_{theor.} = 0.0308$ in and $\sigma_{theor.} = 25.0$ ksi.

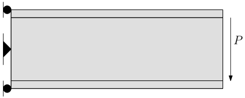
Grid	$\delta/\delta_{theor.}$ (in)	$\sigma_{xx}/\sigma_{theor.}$ (ksi)
11×3	1.045	0.77
21×5	1.025	0.89
31×7	1.016	0.94
41×9	1.012	0.95
51×11	1.010	0.96
61×13	1.009	0.97

Outline

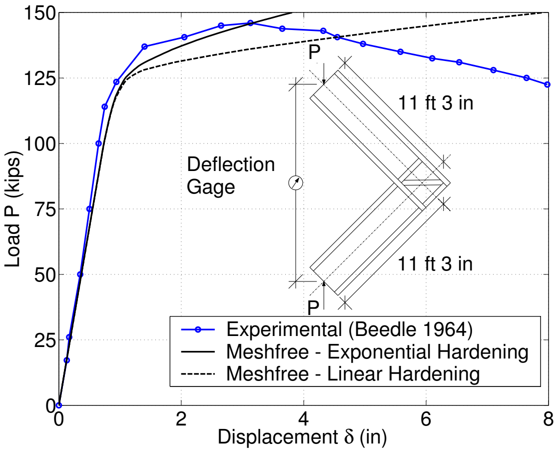
Elasto-plastic cantilever I-beam

J2 plasticity with linear hardening

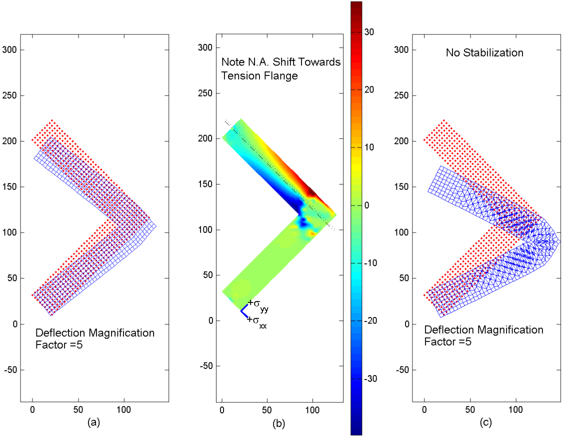




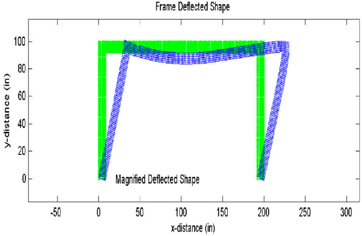
Frame corner connection: Load deflection response

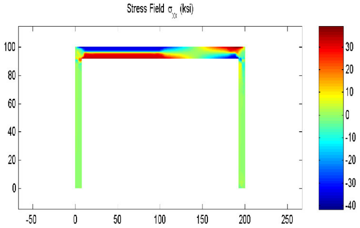


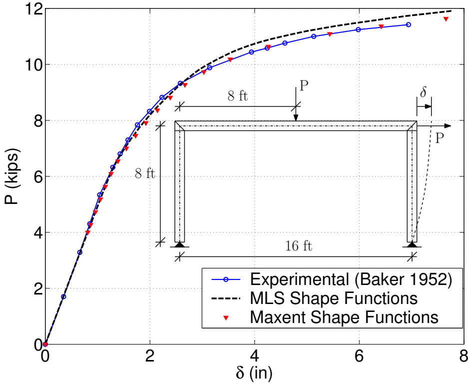
Frame corner connection: Displacement and stress



Inelastic frame analysis







Conclusions

- Demonstrated feasiblity of wide-flange beam analysis under plane stress
- A coupled FE and meshfree method shows promise for inelastic frame analysis
- Demonstrated success of Maxent shape functions
- Further research is ongoing to extend this for large deformations to enable collapse simulations.