Novel Computational Simulation Methodology for Complex Structural System Analysis NSF Grant # CMMI-0826513

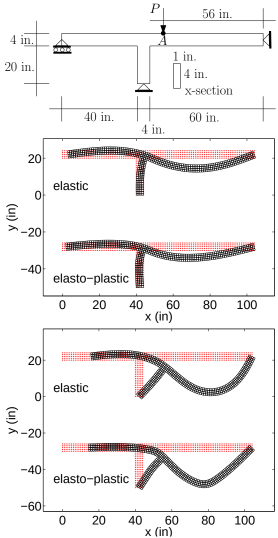
Louie L. Yaw, Sashi Kunnath, N. Sukumar

University of California, Davis
Department of Civil and Environmental Engineering

NSF CMMI Research and Innovation Conference
Honolulu, Hawaii
June 22-25, 2009

Objectives

- Goal to advance large deformation structural analysis to facilitate collapse simulation and failure analysis
- Study feasibility of meshfree technology
- Avoid mesh distortion limitations
- Capture distributed plasticity across beam sections



Key concepts

- Nodal discretization
- Shape functions constructed from nodes
- State variables follow nodes
- Nodes have radius of support (or influence)
- Computationally more expensive than FEM
- Shape functions often lack Kronecker-delta property

MLS Shape Functions Derivation

- Displacement approximation

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_{a=1}^{n} \phi_a(\boldsymbol{x})\boldsymbol{d}_a \equiv \phi^T \boldsymbol{d}$$

- Shape function ϕ_a is of the form (Belytschko et al. 1996)

$$\phi_a(\boldsymbol{x}) = \boldsymbol{P}^T(\boldsymbol{x}_a)\alpha(\boldsymbol{x})w(\boldsymbol{x}_a),$$

$\boldsymbol{P}(\boldsymbol{x}) = \{1\ x\ y\}^T$, a linear basis in two dimensions
$\alpha(\boldsymbol{x})$, a vector of unknowns to be determined
$w(\boldsymbol{x}) \geq 0$, a weighting function

MLS Shape Functions Derivation

- ϕ's must satisfy *reproducing conditions*

$$\boldsymbol{P}(\boldsymbol{x}) = \sum_{a=1}^{n} \boldsymbol{P}(\boldsymbol{x}_a)\phi_a(\boldsymbol{x})$$

- Substitution of ϕ_a into $\boldsymbol{P}(\boldsymbol{x})$ and solving for α yields

$$\alpha(\boldsymbol{x}) = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})$$

- Substituting α into ϕ_a

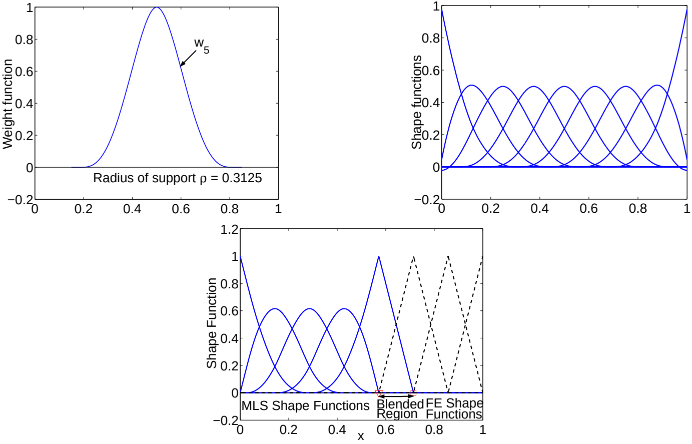
$$\boxed{\phi_a(\boldsymbol{x}) = \boldsymbol{P}^T(\boldsymbol{x}_a)\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x})w(\boldsymbol{x}_a)},$$

where $\boldsymbol{A} = \sum_{a=1}^{n} \boldsymbol{P}(\boldsymbol{x}_a)\boldsymbol{P}^T(\boldsymbol{x}_a)w(\boldsymbol{x}_a)$

Enforcing boundary conditions

- MLS shape functions lack the Kronecker-delta property
- At BC's, MLS shape functions are blended with quadrilateral FE shape functions
- Essential BCs enforced on FE nodes in standard way
- Blending technique is adopted (Huerta and Fernández-Méndez (CMAME 2004))

Weight Function, MLS Shape Functions and Blending



Equilibrium Equations

$$\boldsymbol{f}^{ext} - \boldsymbol{f}^{int} = \mathbf{0},$$

$$\boldsymbol{f}^{ext} = \int_S \phi^T \bar{\boldsymbol{t}}\, dS, \quad \boldsymbol{f}^{int} = \int_V \mathbf{B}^T \boldsymbol{\sigma}\, dV$$

- $\boldsymbol{\sigma} = \boldsymbol{C}\varepsilon = \boldsymbol{C}\mathbf{B}\boldsymbol{d}$, Cauchy stress
- $\bar{\boldsymbol{t}}$, prescribed traction vector
- body forces neglected

Nodal Integration

- Node *a smoothed* strain tensor (Chen et al. (IJNME 2001))

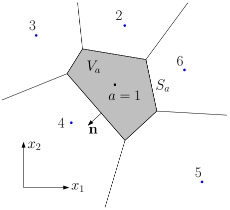
$$\varepsilon_{ij}(\boldsymbol{x}_a) = \frac{1}{2A_a}\int_{V_a} (u_{i,j} + u_{j,i})\, dV = \frac{1}{2A_a}\int_{S_a} (u_i n_j + u_j n_i)\, dS$$

- Strain-displacement relation

$$\boldsymbol{\varepsilon}(\boldsymbol{x}_a) = \sum_{b=1}^{n_N} \mathbf{B}_b(\boldsymbol{x}_a)\boldsymbol{d}_b \equiv \mathbf{B}(\boldsymbol{x}_a)\boldsymbol{d}$$

$$\mathbf{B}(\boldsymbol{x}_a) = [\mathbf{B}_1\ \mathbf{B}_2 \cdots \mathbf{B}_6]$$

$$n_N = \textit{number of neighbors}$$



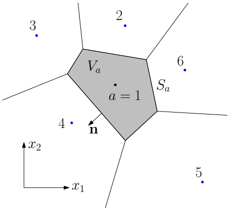
Nodal Integration

- Strain-displacement definitions

$$\boldsymbol{\varepsilon} = [\varepsilon_{11}\ \varepsilon_{22}\ 2\varepsilon_{12}]^T \quad \text{and} \quad \boldsymbol{d}_a = [d_{a1}\ d_{a2}]^T$$

$$\mathbf{B}_b(\boldsymbol{x}_a) = \begin{bmatrix} b_{b1}(\boldsymbol{x}_a) & 0 \\ 0 & b_{b2}(\boldsymbol{x}_a) \\ b_{b2}(\boldsymbol{x}_a) & b_{b1}(\boldsymbol{x}_a) \end{bmatrix}$$

$$b_{bi}(\boldsymbol{x}_a) = \frac{1}{A_a} \int_{S_a} \phi_b(\boldsymbol{x}) n_i(\boldsymbol{x})\, dS$$



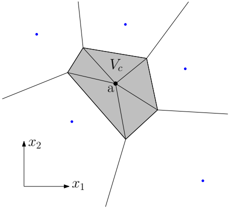
- Nodally integrated stiffness matrix

$$\boldsymbol{K}(\boldsymbol{x}_a) = \mathbf{B}^T(\boldsymbol{x}_a)\mathbf{C}\mathbf{B}(\boldsymbol{x}_a) A_a t_a$$

Stabilization of Nodal Integration (Puso et al. IJNME 2008)

- Integrate each triangular subcell c to obtain $\mathbf{B}^c$
- Stabilize nodally integrated stiffness matrix

$$\boldsymbol{K}^s(\boldsymbol{x}_a) = \boldsymbol{K}(\boldsymbol{x}_a) + \left[\alpha_s \sum_{c \in T_a} (\mathbf{B}(\boldsymbol{x}_a) - \mathbf{B}^c(\boldsymbol{x}_a))^T \mathbf{C}_s (\mathbf{B}(\boldsymbol{x}_a) - \mathbf{B}^c(\boldsymbol{x}_a)) A_c t_a \right]$$

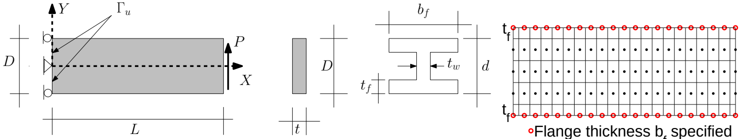


- Stabilize local internal forces:

$$\boldsymbol{f}_\ell^{int} = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma} dV + \alpha_s \sum_{c \in T_a} \left[\int_\Omega (\mathbf{B} - \mathbf{B}^c)^T \mathbf{C}_s (\mathbf{B} - \mathbf{B}^c) \boldsymbol{d}_\ell \, dV_c \right].$$

Meshfree analysis of wide-flange steel sections

- Generate Voronoi diagram based on beam's nodal discretization
- Specify thickness for each Voronoi cell
- Specify web thickness and flange thickness to get wide-flange behavior



(a) Cantilever Beam (b) Rectangular Cross-section (c) *I*-beam

Linear elastic cantilever I-beam

Normalized tip displacement and maximum bending stress

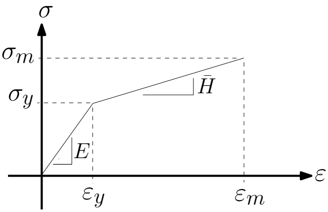
Grid	$\delta/\delta_{theor.}$ (in)	$\sigma_{xx}/\sigma_{theor.}$ (ksi)
11×3	1.045	0.77
21×5	1.025	0.89
31×7	1.016	0.94
41×9	1.012	0.95
51×11	1.010	0.96
61×13	1.009	0.97

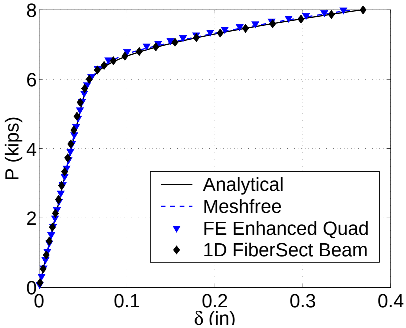
$$\delta_{theor} = 0.0308 \text{ in.}$$

$$\sigma_{theor} = 25.0 \text{ ksi}$$

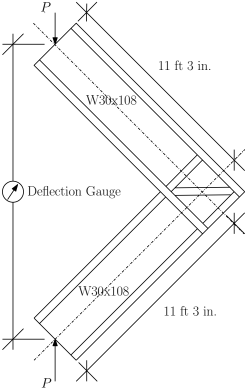
Cantilever I-beam - J2 plasticity, linear hardening

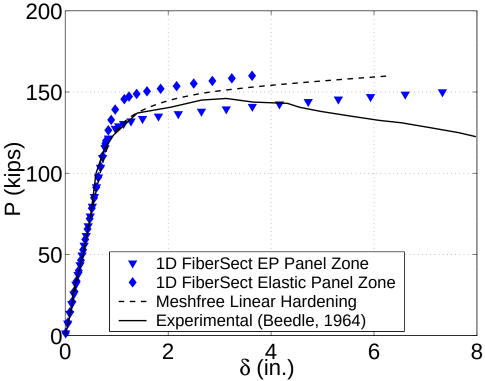
Implicit Newton-Raphson iterations at global level and at constitutive level for J2 plasticity with radial return



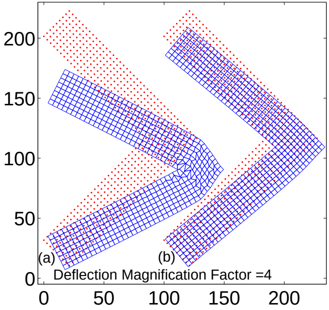


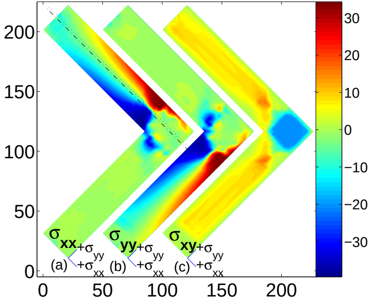
Frame corner connection: Load deflection response





Frame corner connection: Displacement and stress





- (a) No stabilization
- (b) Stabilization

- (a) σ_{xx} (ksi)
- (b) σ_{yy} (ksi)
- (c) σ_{xy} (ksi)

Inelastic frame analysis

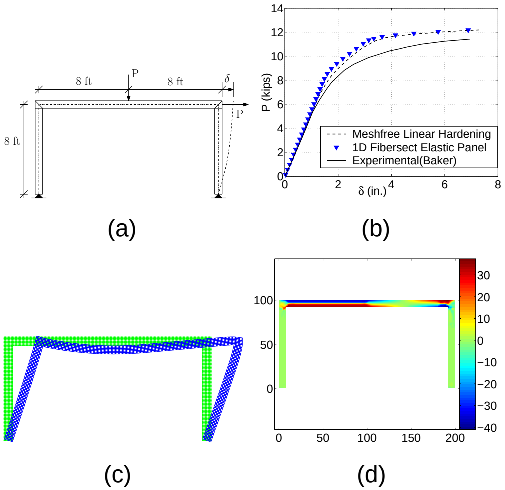
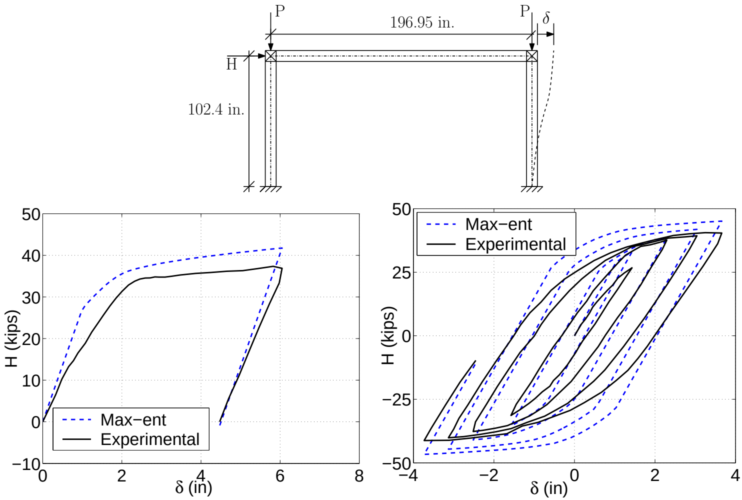


Figure: (a) Frame Loading; (b) Load versus displacement results; (c) Deflected shape; (d) σ_{xx} stress plot (ksi)

Portal frames - monotonic and cyclic loading



(Experimental – Toma et al. J. Constr. Steel Rsrch. 1995)

Conclusions

- Demonstrated feasiblity of wide-flange beam analysis under plane stress
- A coupled FE and meshfree method shows promise for inelastic frame analysis
- Distributed plasticity across beam sections successful
- Future research possibilities
  - meshfree co-rotational formulation
  - concrete beams
  - finite strains
  - Maximum Entropy basis functions

Acknowledgements

- Helpful discussions with Dr. Michael Puso, Lawrence Livermore National Laboratory
- Study leave funding from Walla Walla University
- National Science Foundation contract grant # CMMI-0826513