The Bergeron Method

A Graphic Method for Determining Line Reflections in Transient Phenomena

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TEXAS INSTRUMENTS

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Abstract

The user of modern, fast, logic systems must consider the effect of line reflections very closely. This report describes the Bergeron method of determining line reflections, then illustrates some typical examples of its use and the results it produces.

1 Introduction

The engineer designing fast digital systems must consider the response of electric signals on circuit lines very closely to avoid interference caused by signal distortions. Applications of familiar mathematic methods of line-transmission theory are limited in determining line reflections because the nonlinear input and output characteristics of digital circuits are difficult to represent in mathematical terms.

In transient phenomena on lines with defined characteristic impedances, it is simple to show the response of current and voltage as a function of time, at the beginning and end of a circuit line, in graphic form. This method is particularly suitable for the nonlinear line terminations used in digital engineering.

2 Derivation of Method

A transient phenomenon propagates (see Figure 1) along a line (A, B) in the form of a forward (v_f, i_f) and reflected (v_r, i_r) voltage and current waves as follows:

$$v_{(x,\,t)} = v_f + v_r \tag{1}$$

$$i_{(x,\,t)} = i_f + i_r \tag{2}$$

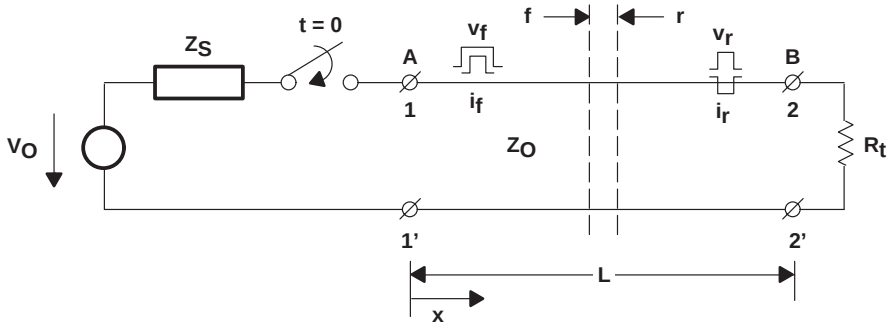

Figure 1. Wave Propagation Along a Line

The ratio of voltage to current for the traveling waves is the characteristic impedance, and the negative sign of reflected waves stems from the reversed current sense of i_r as follows:

$$Z_O = \frac{v_f}{i_f} \tag{3}$$

$$Z_O = \frac{v_r}{-i_r} \tag{4}$$

Equations 1 through 4 produce:

$$v + Z_O \times i = v_f + v_r + Z_O \times (i_f + i_r) = 2v_f(x - v \times t) \tag{5}$$

$$v - Z_O \times i = v_f + v_r - Z_O \times (i_f + i_r) = 2v_r(x + v \times t) \tag{6}$$

Whereby:

$$v = \frac{1}{\sqrt{L'C'}} \quad \text{(velocity of traveling waves)} \tag{7}$$

$$Z_O = \sqrt{\frac{L'}{C'}} \quad \text{(characteristic impedance)} \tag{8}$$

Where:

L' = inductance per unit of line length (nH/m)
C' = capacitance per unit of line length (pF/m)
$(x - v \times t)$ = delay parameter for forward wave (f)
$(x + v \times t)$ = delay parameter for reflected wave (r)

If an observer moves with the forward or reflected wave (i.e., observer and wave are equally fast), Figure 2 applies to the delay parameters.

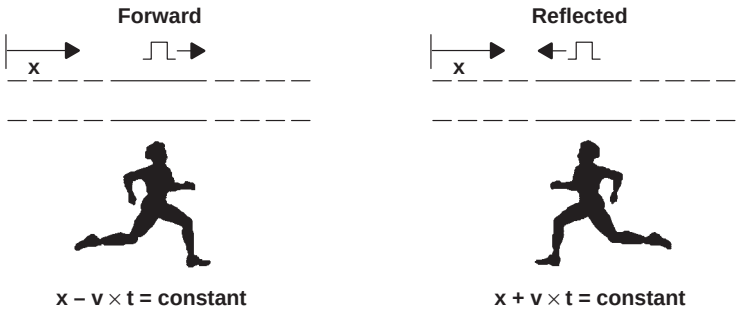

Figure 2. Wave Propagation

If the delay parameters are constant, this also applies to their functions as follows:

Forward

$$f(x - v \times t) = v_f(x - v \times t) = \text{constant} \tag{9}$$

$$f(x - v \times t) = v + Z_O \times i = \text{constant} \tag{10}$$

Reflected

$$f(x + v \times t) = v_f(x + v \times t) = \text{constant} \tag{11}$$

$$f(x + v \times t) = v - Z_O \times i = \text{constant} \tag{12}$$

The straight lines of the functions are shown in Figure 3.

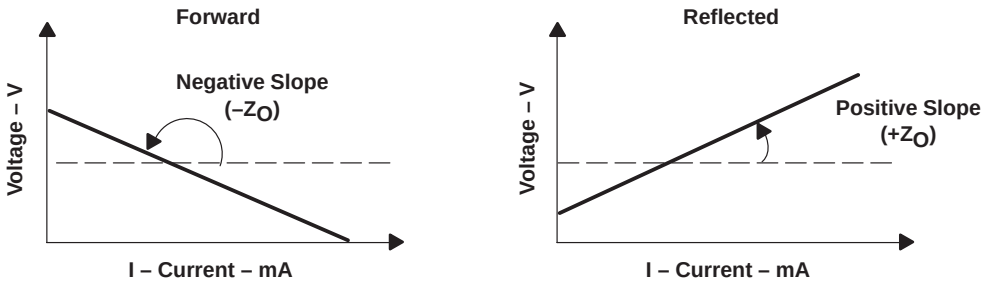

Figure 3. Function of Current and Voltage

The constants of the functions define the location of the straight lines. They are determined by the boundary conditions of the transient phenomenon. In movement with the wave, all operating values for current and voltage of the straight lines are passed through. The values for current and voltage, at the beginning or end of the line, are given by the points of intersection of the conditional straight lines with the characteristics of the line terminations (impedance and generator characteristics).

3 Ideal Voltage Source

3.1 Turning On an Ideal Voltage Source (Open Circuit at End of Line)

To begin with, consider a case in which an ideal voltage source (V_O) (source impedance = 0 Ω) is connected to a loss-free line with the characteristic impedance (Z_O) unterminated at the end (open circuit). Of course, this case is not found in actual practice because there are neither ideal voltage sources nor loss-free lines. Nevertheless, it clearly shows the influence of line reflections on signal shape in extreme cases.

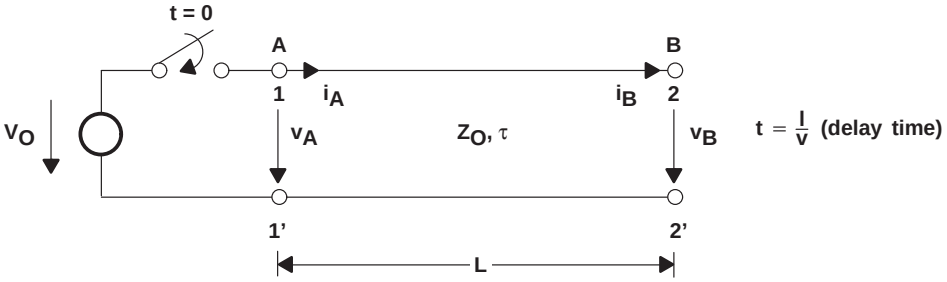

Figure 4. Ideal Voltage Source and Open-Circuit Line

To obtain the potential gradients at the beginning and end of the line, first enter (as shown in Figure 5) the generator characteristic (parallel to the I axis because source impedance = 0 Ω) and the characteristic of the line termination (coincides with the V axis because the line is open circuit). Then determine the state of the line before switching. Enter the values for voltage and current at the end of the line at the point in time t < 0 (in this case, the intersection of the V and I axes). These values define the starting point of reflection to the beginning of the line (Bergeron straight line) as follows:

Starting point

$$v_{B(t<0)} = 0 \quad \textbf{(13)}$$

$$i_{B(t<0)} = 0 \quad \textbf{(14)}$$

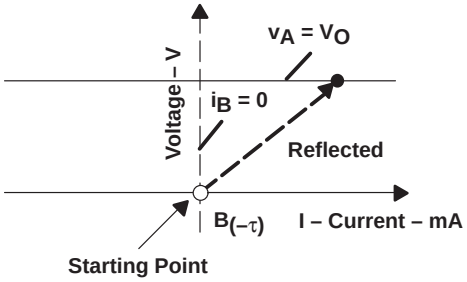

Figure 5. Voltage Current Diagram

Through this point draw a straight line with the slope Z_O (see Figure 6). The intersection of the straight line with the generator characteristic produces the voltage and current value at the beginning of the line at the point in time t = 0.

Ideal voltage source

$$v_A = V_O \quad \textbf{(15)}$$

$$i_A = \frac{V_O}{Z_O} \quad \textbf{(16)}$$

Through this point again draw a straight line with the slope $-Z_O$ (see Figure 6). The intersection of the line with the characteristic of the line termination produces the voltage and current value at the end of the line, when the electric wave has reached the end of the line. The line is open circuit, so the voltage that appears at its end is twice that which is at the beginning of the line (the current is 0). To continue, draw straight lines with the slope Z_O and $-Z_O$ through the respective points that have been defined. This obtains the voltage and current values at the end of the line at the points in time 1τ, 3τ, 5τ, etc., plus the corresponding values for the beginning of the line at the points in time 0τ, 2τ, 4τ, etc. In this particular case, the straight lines meet to form a parallelogram, which means that the line is not in a stationary state at any time.

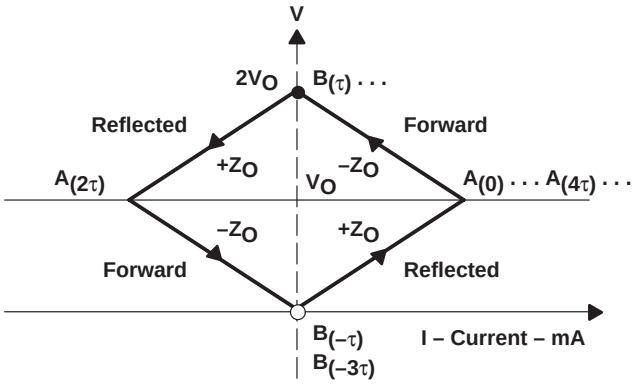

Figure 6. Bergeron Diagram

Current and voltage at points A and B, because of the causality of time, cannot change until a wave has been forwarded and reflected (2τ). Consequently, current and voltage take on a rectangular form (staircase, pulses). Since no energy is consumed at the end of the line, the wave is fully reflected. The forward and reflected waves are superimposed on one another and add to form double the amplitude $2 \times V_O$. The source impedance of the ideal voltage source (0 Ω) also prevents breakdown of the wave energy. This case results in an undamped rectangular oscillation at the end of the line with amplitude $2 \times V_O$ and period 4τ (see Figure 7).

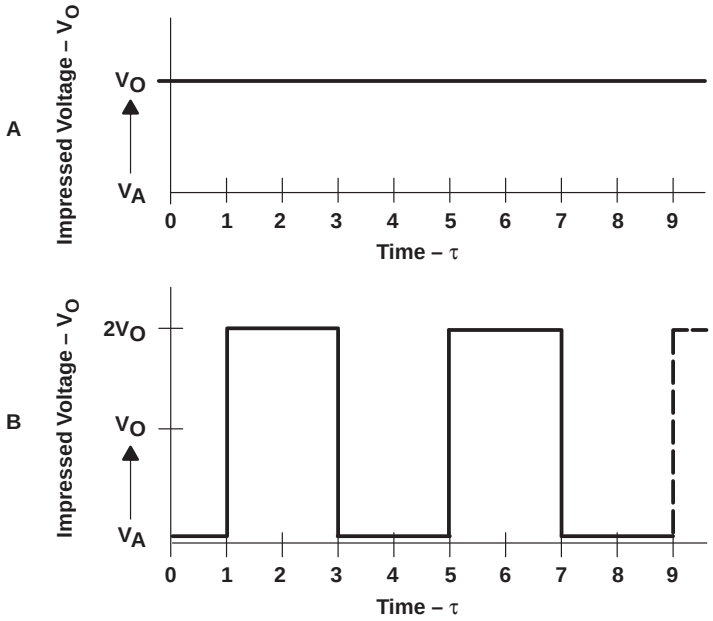

Figure 7. Potential Gradient at Beginning and End of Line

3.2 Turning On an Ideal Voltage Source (Short Circuit at End of Line)

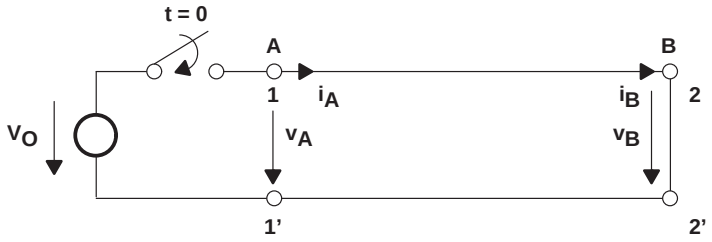

Figure 8. Ideal Voltage Source and Short-Circuited Line

The procedure is the same as in the previous example. First, establish the boundary conditions as follows:

A:

$$v_{A(t=0)} = V_O \tag{17}$$

B:

$$v_{B(t=0)} = 0 \tag{18}$$

Before turn-on, the line is without energy. For $t < 0$, therefore:

Starting point

$$v_{B(t<0)} = 0 \tag{19}$$

$$i_{B(t<0)} = 0 \tag{20}$$

Because the line is short-circuited at the end, the characteristic of the terminating impedance is coincident with the horizontal abscissa. Proceeding according to the method previously described and drawing the straight lines with the slope Z_O and $-Z_O$ in Figure 9, a zigzag line is obtained following the positive sense of current. There is no attenuation, so the current increases in steps towards infinity (see Figure 11).

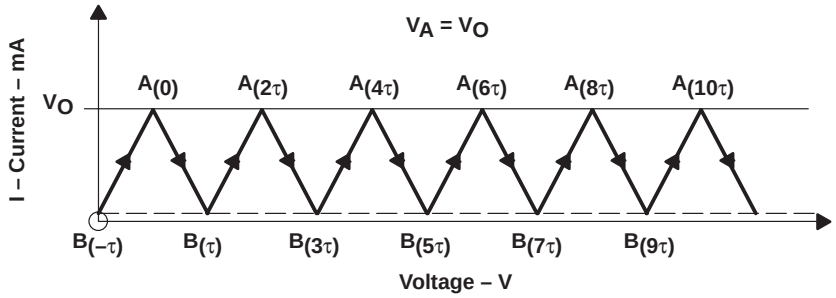

Figure 9. Bergeron Diagram for Short-Circuited Line

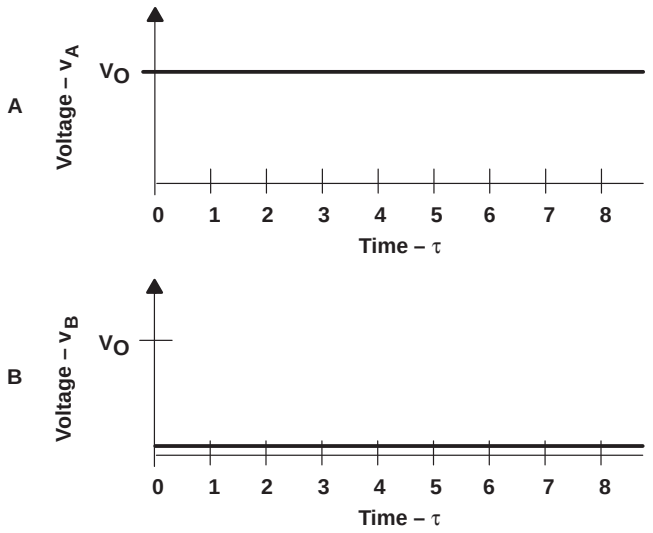

Figure 10. Potential Gradient for Short-Circuited Line at Beginning and End of Line

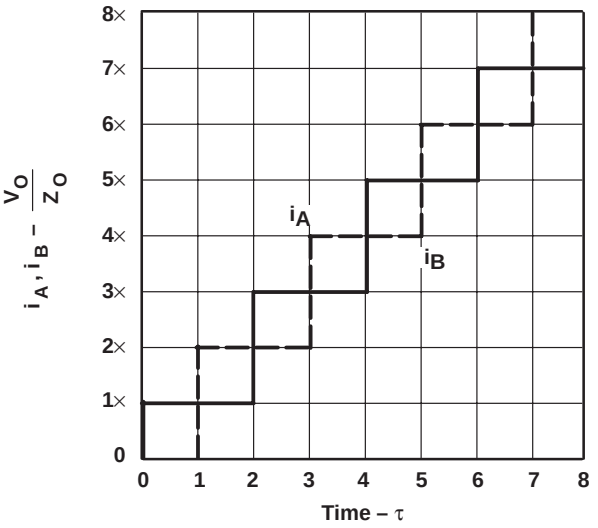

Figure 11. Current Form for Short-Circuited Line at Beginning and End of Line

4 Real Voltage Sources

4.1 Turning On a Real Voltage Source

4.1.1 Terminated Line

The examples described previously have no practical significance because there are no ideal voltage sources and the lines themselves are not loss-free either, although these losses may be neglected in the cases looked at here. Also, lines over which a signal is transmitted are terminated at the end in some form or another, if only by the input impedance of a receiver. Therefore, in what follows, the line reflections are examined that appear when a voltage source driving a line has a certain source impedance. The major cases encountered in practice are shown in Figure 12.

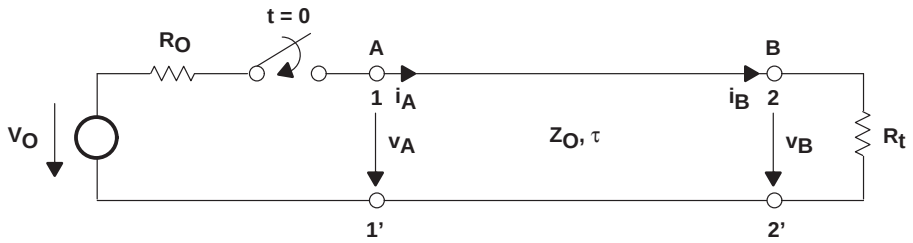

Figure 12. Real Voltage Source and Terminated Line

In a steady-state condition (i.e., t = ∞), the characteristics of the line are ineffective. Then the voltage that appears is a result of the voltage division by the voltage source impedance and the terminating impedance at the end of the line (see Figure 13).

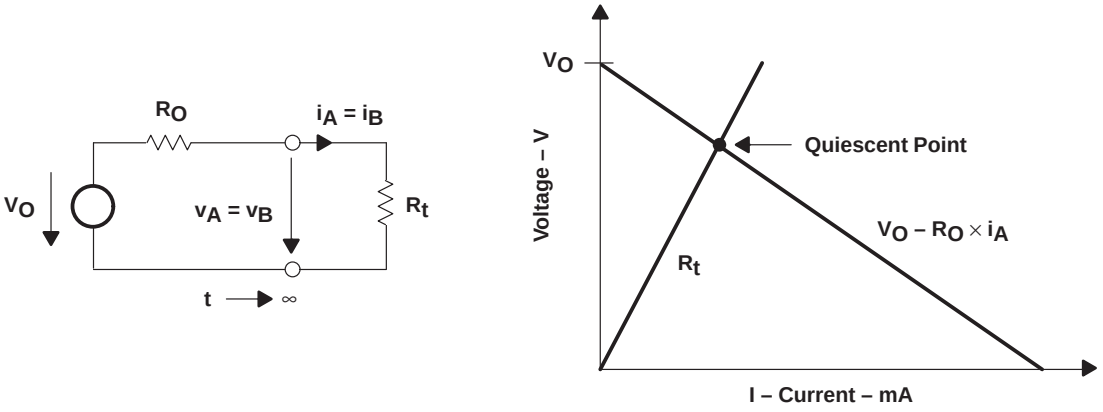

Figure 13. Line in Steady-State Condition

Determining the line reflections by the Bergeron method is performed in a manner similar to that previously described. At time t < 0 (switch open), the current and voltage on the line are 0. This value produces the starting point in Figure 14. The wave plan shows a zigzag form with the slope $\pm Z_O$ between the characteristics of the two line terminations. The zigzag line ends at the quiescent point. The condition on the line then is steady state, and there are, again, dc relationships.

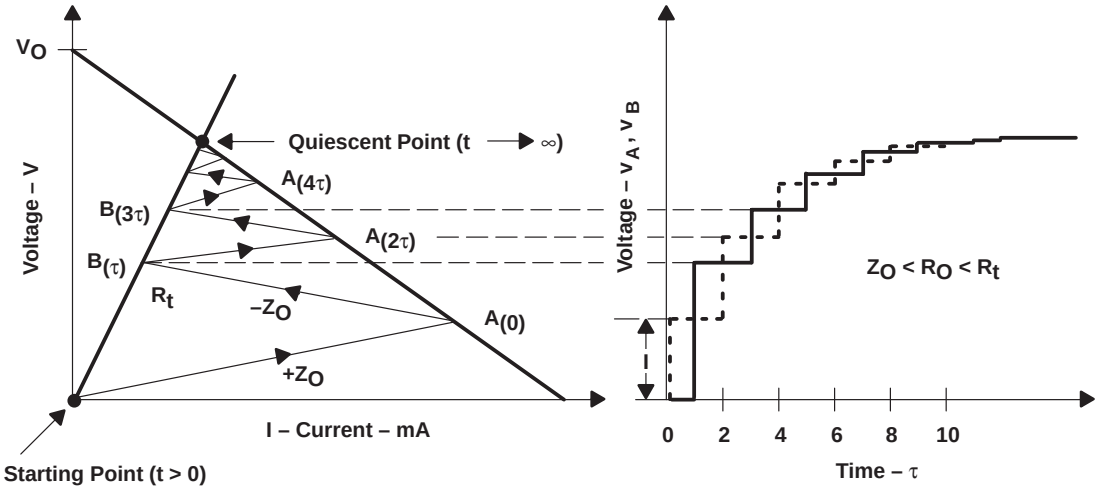

Figure 14. Line Reflections for Terminated Line

4.1.2 Unterminated Line

With an open-circuit line (i.e., $R_t = \infty$), the straight line for the impedance coincides with the V axis of the diagram. In this case, too, the voltage slowly builds up to its final value. However, the line is not loaded at the end, so the final value of the amplitude is equal to the open-circuit voltage of the generator V_O.

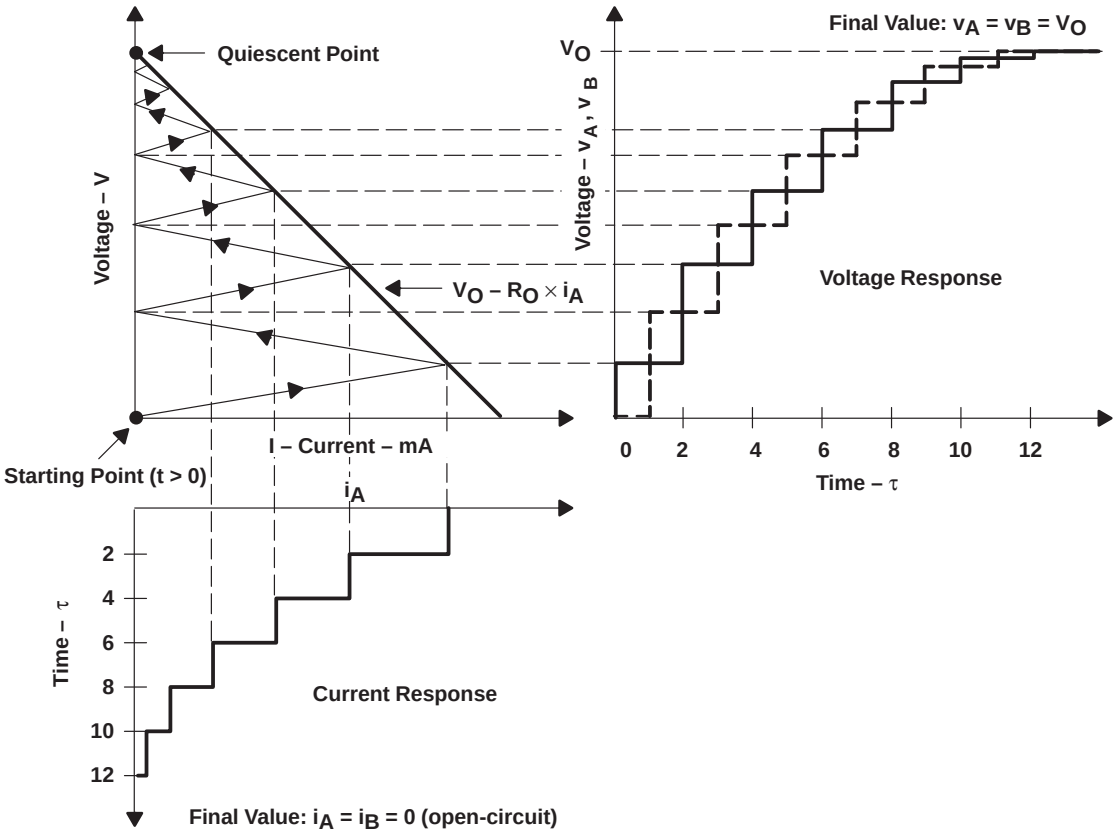

Figure 15. Voltage and Current for Open-Circuit Line

In the examples shown in Figures 14 and 15, the voltage reaches the same amplitude during the time $0 < t < 2\tau$ at the beginning of the line, despite different line terminations. This is because when the switch is closed, the generator only sees the line impedance Z_O that is characteristic of the line for twice the delay time, no matter how the line is terminated.

Observing the reflections shown in Figures 14 and 15 on an oscilloscope with a low cutoff frequency, the risers are obliterated. Consequently, it seems as if the voltage would approach its final value in the form of an e-function. In other words, it appears to the observer that only the capacitance of the connected line is effective. This leads to the wrong conclusion that the connected line would behave like a large capacitive load.

Source impedance of the generator R_O smaller than the characteristic impedance Z_O leads to what is called overshoot at the end of the line. Figures 16 and 17 show the nature of the reflections for a terminated and an open-circuit line.

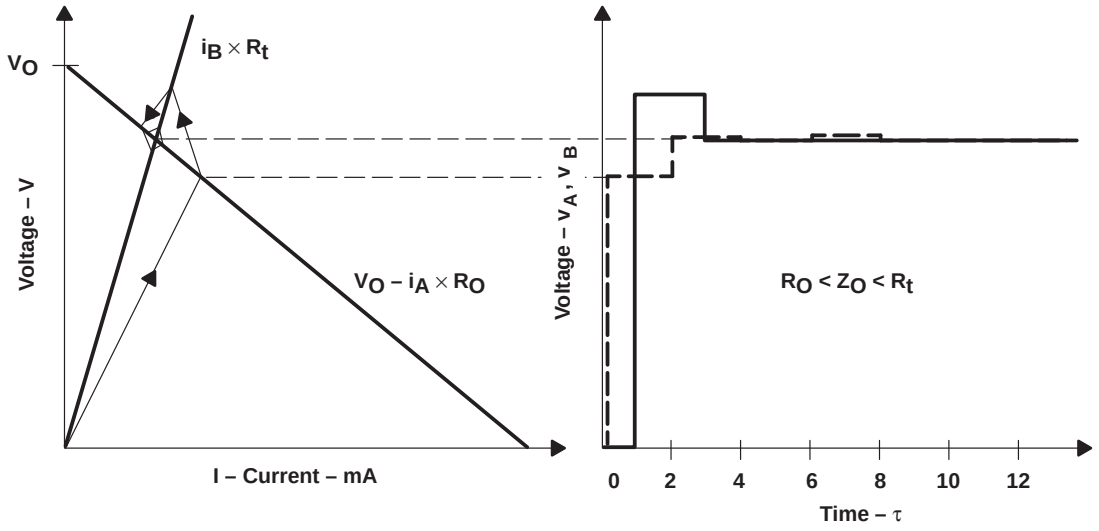

Figure 16. Terminated Line ($R_O < Z_O < R_t$)

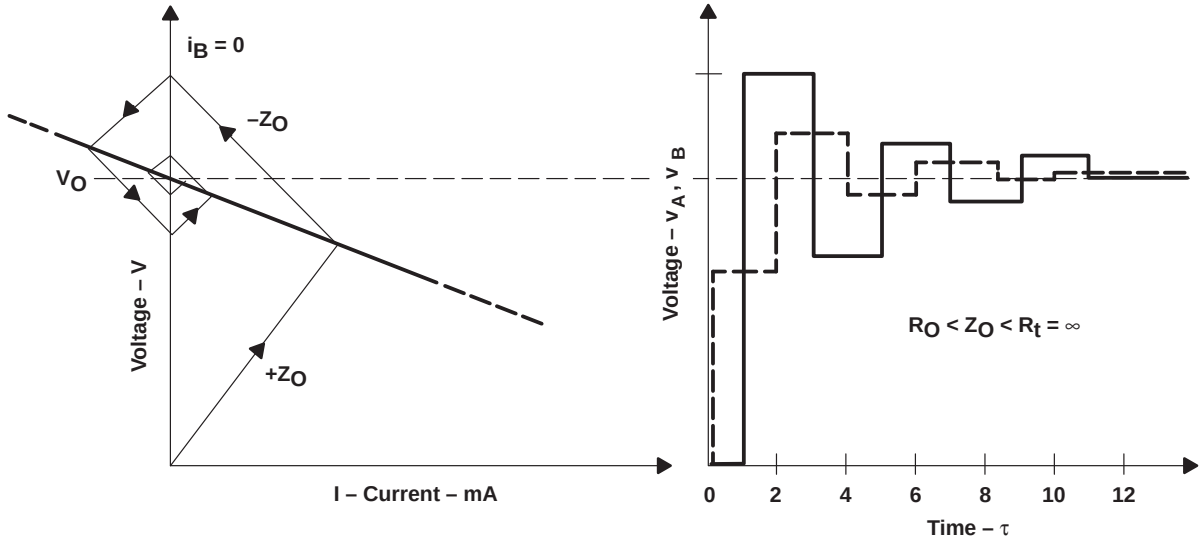

Figure 17. Open-Circuit Line ($R_O < Z_O < R_t$)

Characteristic impedance Z_O greater than the terminating impedance leads to overshoot at the beginning of the line (see Figure 18).

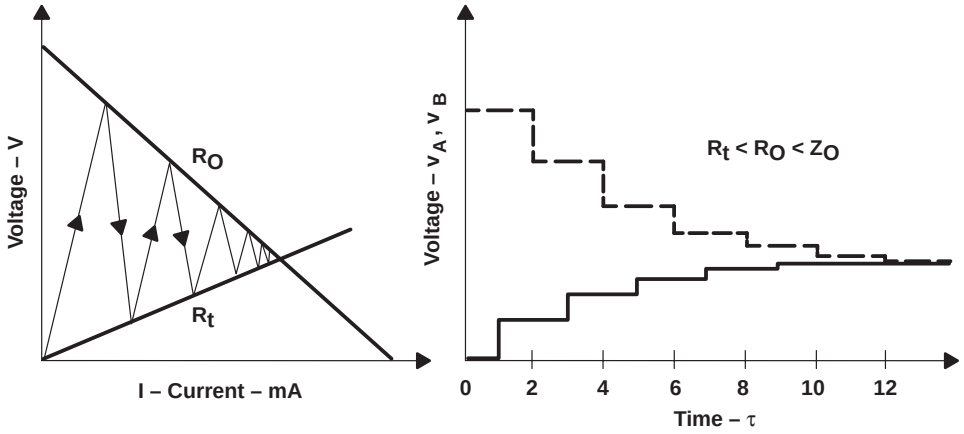

Figure 18. Terminated Line ($R_O < Z_O$)

4.1.3 Matching

Line reflections are avoided if the line is terminated with the characteristic impedance. It is possible to terminate a line either at the beginning, by matching the generator source impedance (shown in Figure 19), or at its end with an appropriate impedance (as shown in Figure 20). In both cases, reflections at the end of the line are avoided. However, if a line is terminated by matching the generator source impedance, there will be reflections at the beginning of the line.

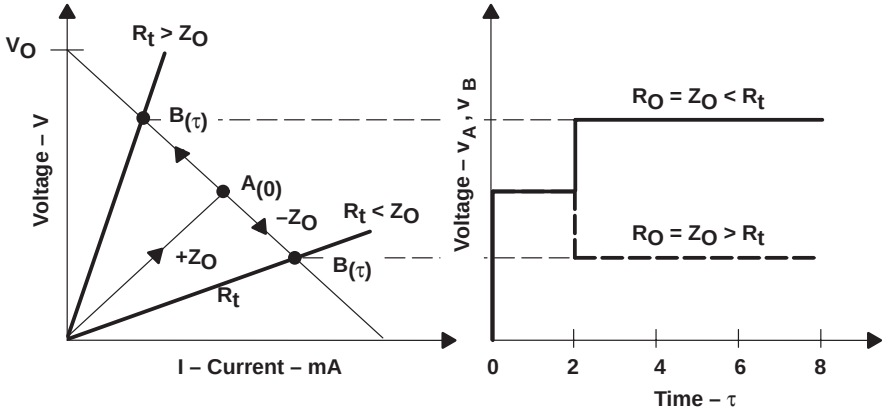

Figure 19. Line Termination by Matching Generator Source Impedance

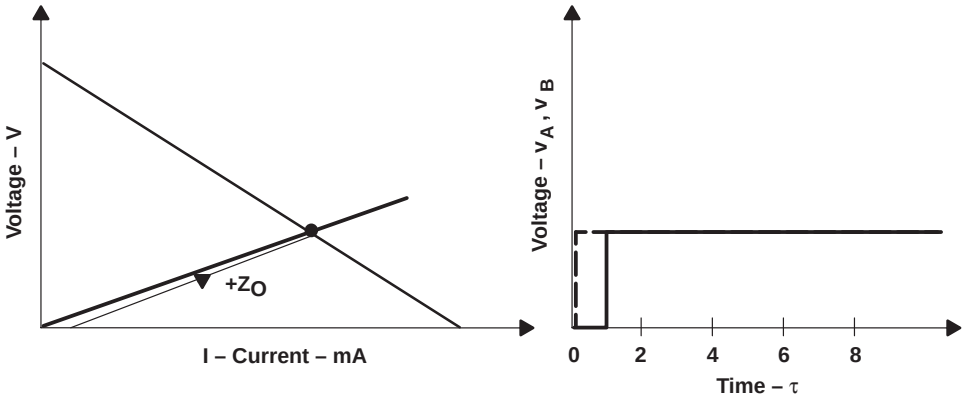

Figure 20. Line Termination by Terminating Impedance at the Line End

4.2 Turning Off a Line

The nature of the reflections on a line, when the voltage source is turned off (see Figure 21), can also be examined by the method previously described. If the switch at the generator output is opened, the energy stored in the capacitance of the line must be broken down and the voltage will not immediately fall to zero.

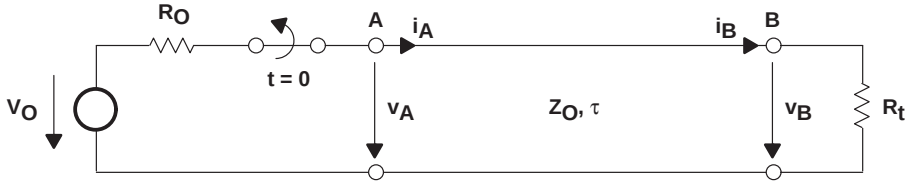

Figure 21. Turning Off a Line

The starting point for the Bergeron method is determined by the state at the point in time t < 0 as follows:

$$v_{B(-\tau)} = \frac{R_t}{R_O + R_t} \times V_O \quad \textbf{(21)}$$

$$i_{B(-\tau)} = \frac{V_O}{R_O + R_t} \quad \textbf{(22)}$$

Boundary condition (t = 0)

A:

$$i_A = 0 \quad \textbf{(23)}$$

B:

$$v_B = i_B \times R_t \quad \textbf{(24)}$$

Figure 22 shows the corresponding Bergeron diagram and the resulting potential gradient.

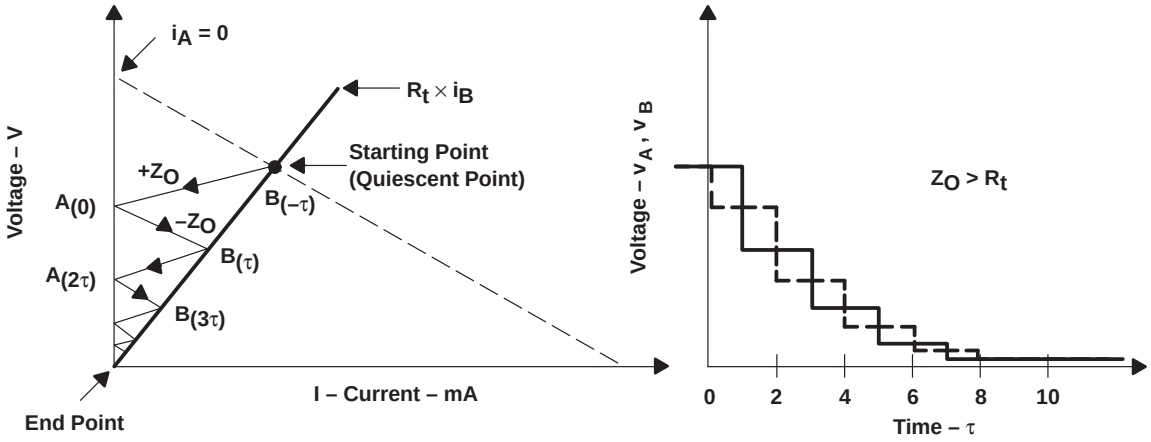

Figure 22. Line Reflections When Turning Off a Line

If the line is terminated at the end with an impedance $R_t = Z_O$, the voltage immediately swings to zero.

4.3 Switching a Line

The outputs of digital circuits are generally push-pull outputs. This means that when an output is switched, either a high voltage or a low voltage is connected to the line across the source impedance of the output stage (see Figure 23).

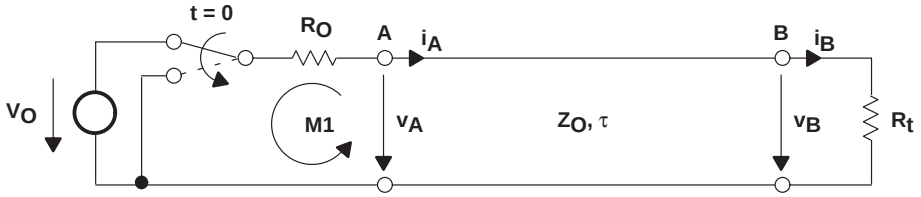

Figure 23. Switching a Line

At the point in time t = 0, the line is short-circuited on the input side across the source impedance of the generator. Before the switching, the same dc values apply as in the previous example.

Boundary condition

A:

$$v_A = -R_O \times i_A \text{ (loop M1)} \quad \textbf{(25)}$$

B:

$$v_B = R_t \times i_B \quad \textbf{(26)}$$

At the beginning of the line and at the end there are finite impedances, so the wave energy and, thus, the voltages V_A and V_B can break down faster than in the previous example.

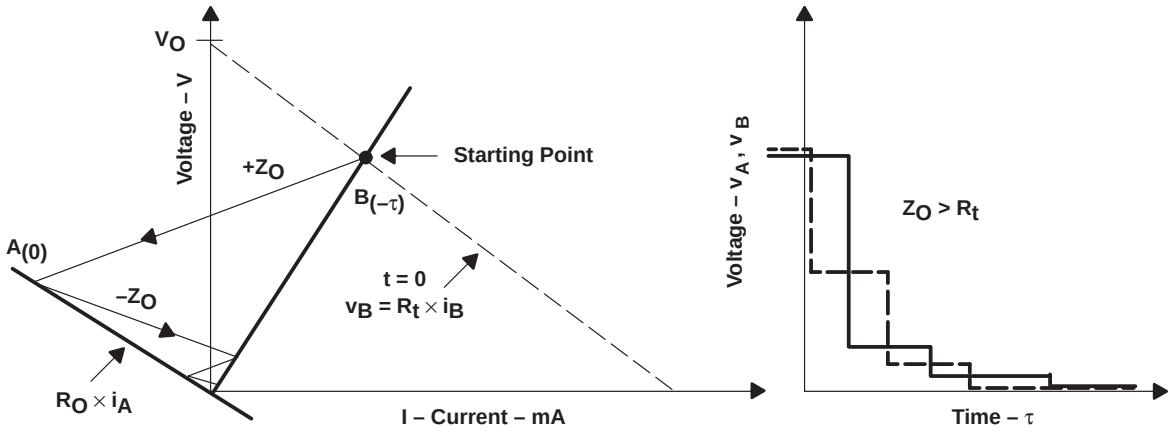

Figure 24. Line Reflections When Switching a Line ($Z_O > R_t$)

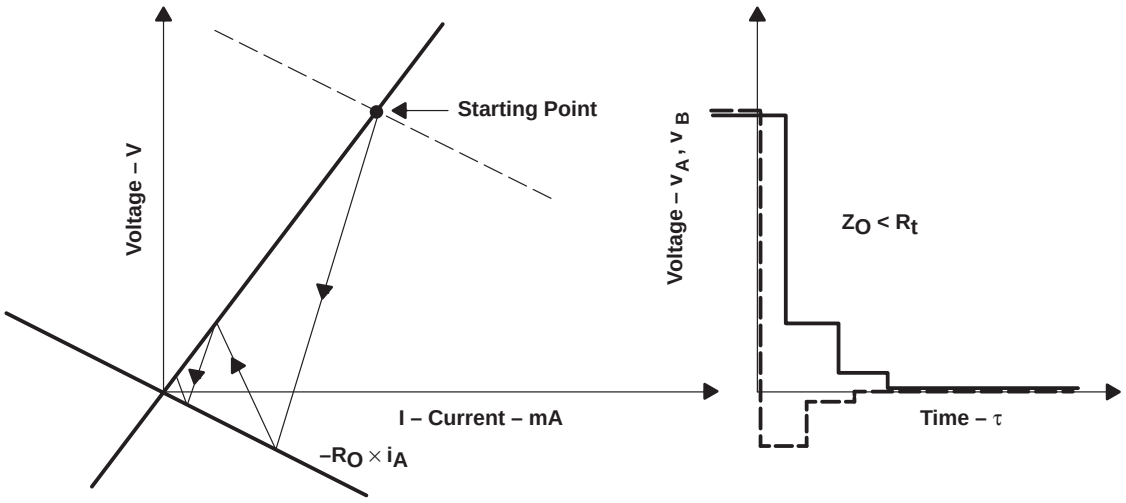

Figure 25. Line Reflections When Switching a Line ($Z_O < R_t$)

5 Bergeron Method With Nonlinear Line Terminations

As mentioned previously, the method can easily be applied for nonlinear characteristics exhibited by the inputs and outputs of TTL circuits. The senses shown in Figure 26 apply to currents and voltages.

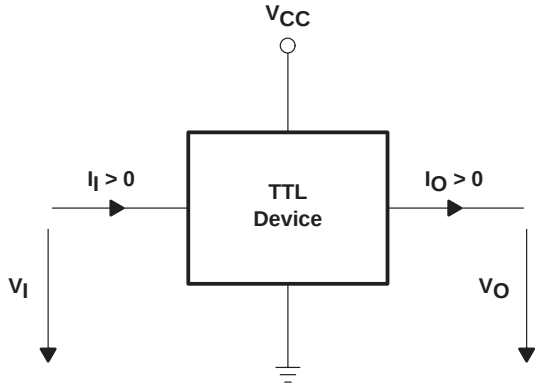

Figure 26. Definition of Current and Voltage Senses

Figure 27 shows the typical input and output characteristics of an AS/TTL circuit. A high output responds like a voltage source with an open-circuit voltage of about V_{OH} = 3.5 V and a source impedance of $R_O \approx 30\ \Omega$. In the low condition with positive voltages, the very low collector series resistance (≈3 Ω) is primarily effective, while negative voltages are limited by a Schottky diode at the output. The input characteristic is determined in the positive region by the high input impedance of the circuit (several kΩ). Again, negative voltages are limited by a Schottky diode (clamping diode).

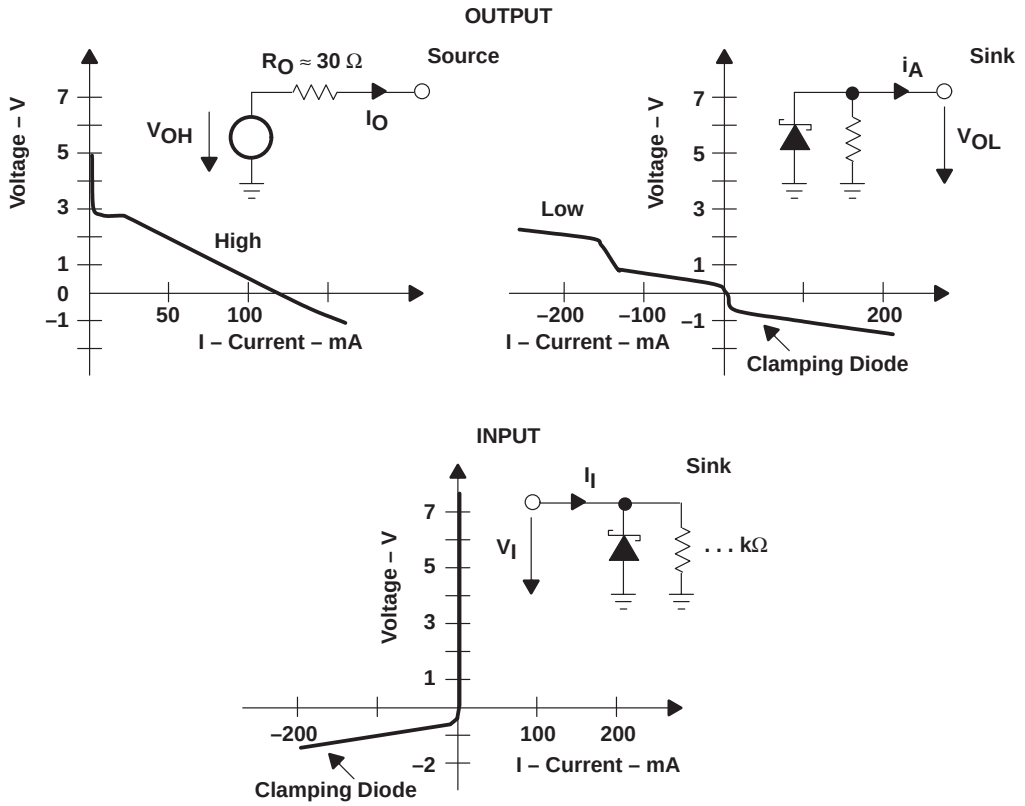

Figure 27. Input and Output Characteristics of AS/TTL Circuits

5.1 Gate Configuration Without Matching

Figure 28 shows a typical transmission line when two gates are connected on a circuit board.

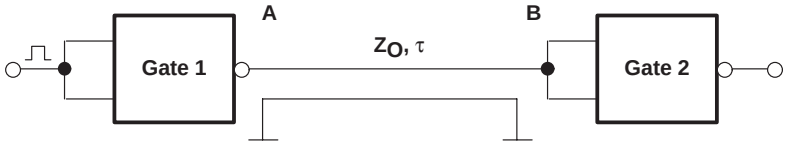

Figure 28. Transmission Line Without Matching

For the above configuration, all the necessary characteristics are entered into the V/I diagram (see Figure 29). The intersections of the input characteristic with the output characteristics are the steady-state conditions for high or low, and thus, the starting and finishing points of the Bergeron straight lines.

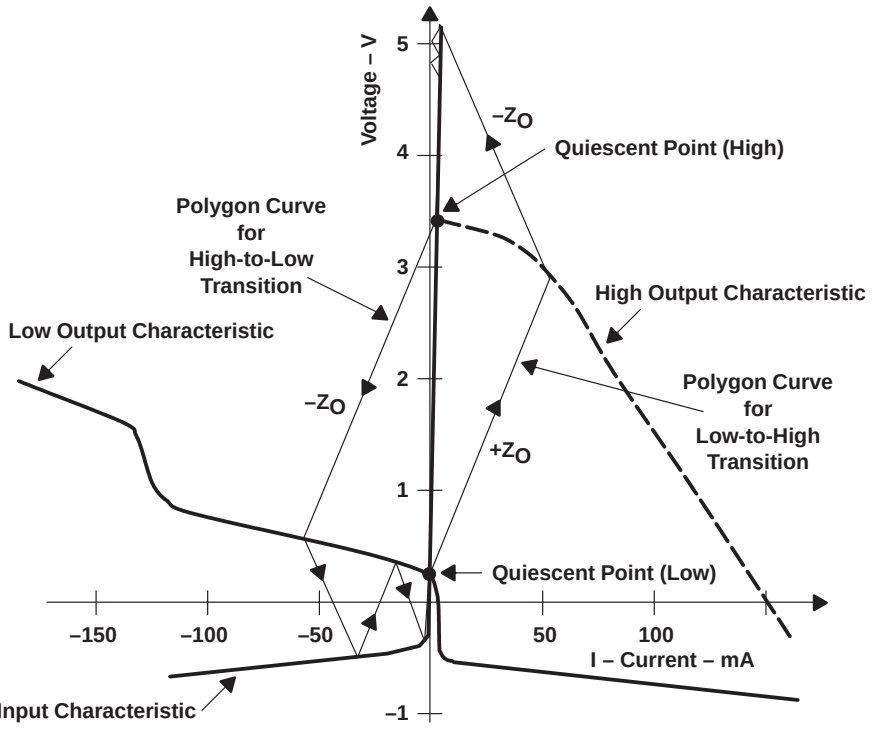

Figure 29. Bergeron Diagram for Transmission Line Between SN74AS000 TTL Circuits

5.1.1 Low-to-High Transition

Because of the low loading of the output by the line ($R_O < Z_O$), the voltage at the beginning of the line immediately rises to a value of about 2.5 V. The end of the line is terminated by the high input impedance of gate 2, thus, there is virtually a doubling of voltage. In this voltage region, the output impedance of gate 1 is also very large, so the voltage is slow in building up to its final value (see Figure 30).

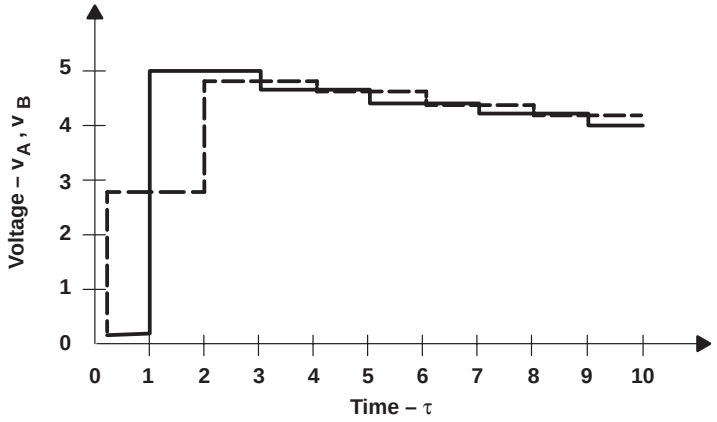

Figure 30. Line Reflections (Low-to-High Transition)

5.1.2 High-to-Low Transition

At the output of gate 1, the voltage initially jumps to a value of 1 V. At the end of the line, the negative overshoot is very much limited by the diode characteristic of the input circuit, meaning that the line reflections rapidly decay (see Figure 31).

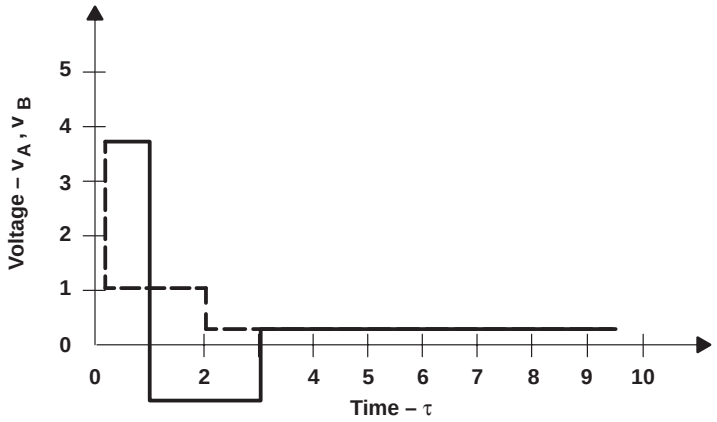

Figure 31. Line Reflections (High-to-Low Transition)

Figure 32 shows the oscillogram recorded for this data-transmission line. As shown, the actual response of the circuit coincides with that predicted using the Bergeron method.

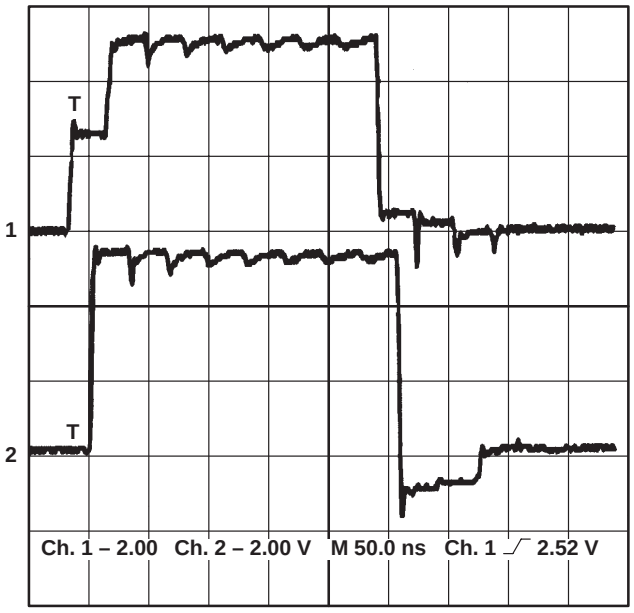

Figure 32. Measured Line Reflections (Line Length = 3 m, Z_O = 50 Ω)

5.2 Transmission Lines With Line Termination

Generally, lines in a TTL system do not have to be terminated. As shown previously, the reflections are usually adequately damped by the output impedance of the circuits (matching to the beginning of the line) or by clamping diodes (short-circuiting of the reflected energy). However, in certain applications, e.g., large bus systems, it may be necessary to provide for further damping of reflections by an additional termination of the line. The first possibility is that of terminating the end of the line with a resistance (see Figure 33).

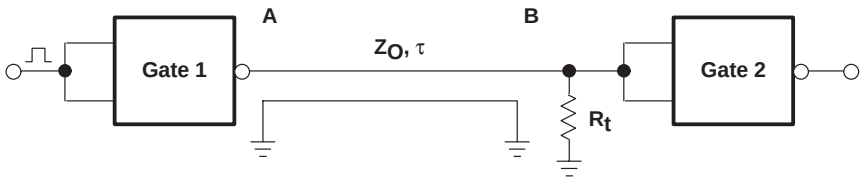

Figure 33. Termination of Line With a Resistor

Figure 34 shows the new characteristic of the line termination composed of the input characteristic of gate 2 and the parallel resistance R_t.

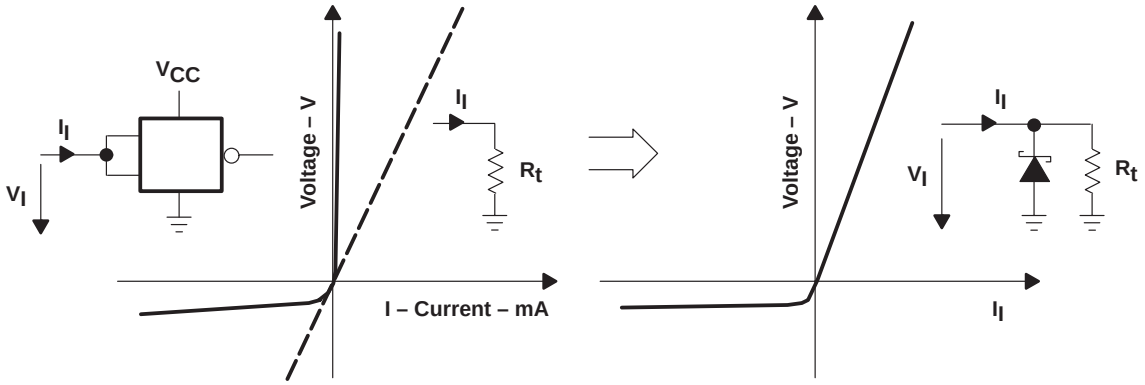

Figure 34. Characteristic of Line Termination

Figure 35 shows the signal shapes resulting on the line with matching ($R_t = Z_O$). As expected, reflections are completely avoided in this circuit. However, a drawback in this circuit is the high current that flows across the resistor, and the associated power consumption.

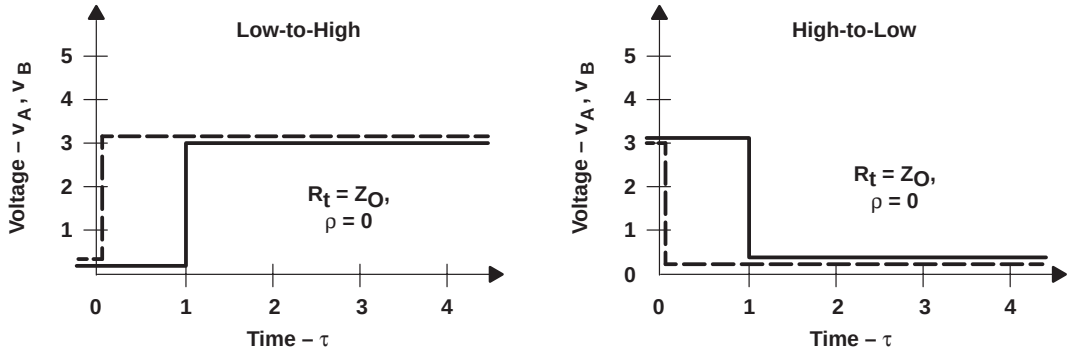

Figure 35. Signal Response for Matched Line

5.3 Split Termination

To reduce the dissipation in the terminating impedance, the line termination shown in Figure 36 is often used in TTL systems. In this case, the termination is split into two series-connected resistors (R1 and R2), thus, a split termination.

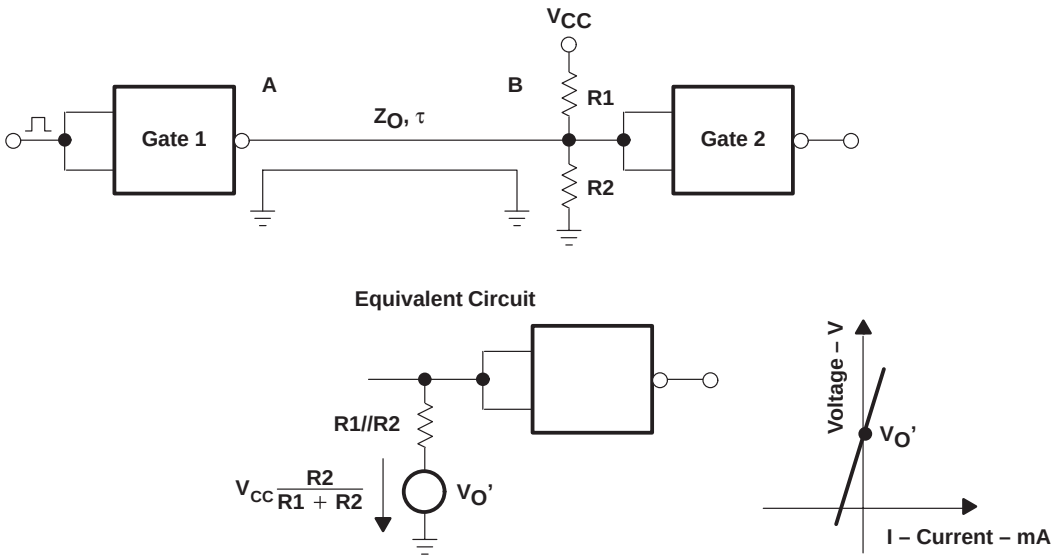

Figure 36. Line Termination by Voltage Divider (Split Termination)

Figure 37 shows the new characteristic of the line termination. The signal shapes are identical to those in Figure 34, with the exception that the logic levels are somewhat higher. This applies in particular to the high level, which, in this circuit, is primarily determined by the open-circuit voltage of the voltage divider. The voltage divider is dimensioned according to the following equations:

$$\frac{R1 \times R2}{R1 + R2} = Z_O \tag{27}$$

$$\frac{R2}{R1 + R2} \times V_{CC}\text{min} > 2.4\ V \tag{28}$$

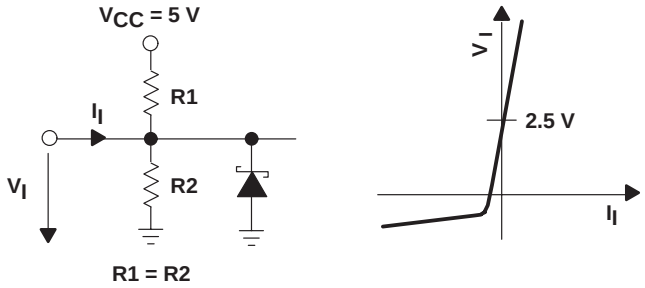

Figure 37. Characteristic of Line Termination by Voltage Divider (Split Termination) and Clamping Diode

5.4 Nonideal Matching

The good noise margin of TTL circuits allows a certain amount of line reflection. Ideal matching is, consequently, superfluous. Thus, termination can be a much higher impedance ($R_t > Z_O$) and power consumption can be reduced appreciably. TTL circuits provide a typical noise margin (N) as follows:

$$N = \frac{V_{TH} - V_{OL}}{V_{OH} - V_{OL}} = \frac{1.4\ V - 0.4\ V}{3.5\ V - 0.4\ V} \approx 0.35 \tag{29}$$

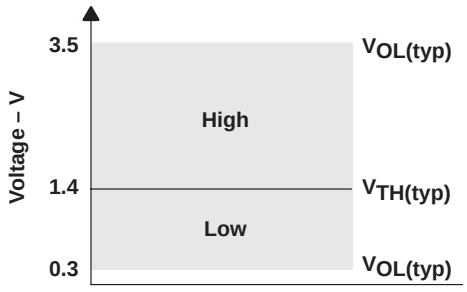

Figure 38. Typical Noise Margin of TTL Circuits

Spending 50% of the noise margin for line reflections, the acceptable reflection factor will be:

$$\rho = \frac{N}{2} \approx 0.2 \text{ (safety factor)} \quad \textbf{(30)}$$

With a reflection coefficient of $\rho = 0.2$, interference-free data transmission is ensured. From the equation for the reflection coefficient, there follows:

$$\rho = \frac{R_t - Z_O}{R_t + Z_O} = 0.2 \quad \textbf{(31)}$$

$$R_t = \frac{1.2}{0.8} \times Z_O = 1.5 \times Z_O \quad \textbf{(32)}$$

Equation 27 alters as follows:

$$\frac{R1 \times R2}{R1 + R2} = 1.5 \times Z_O \quad \textbf{(33)}$$

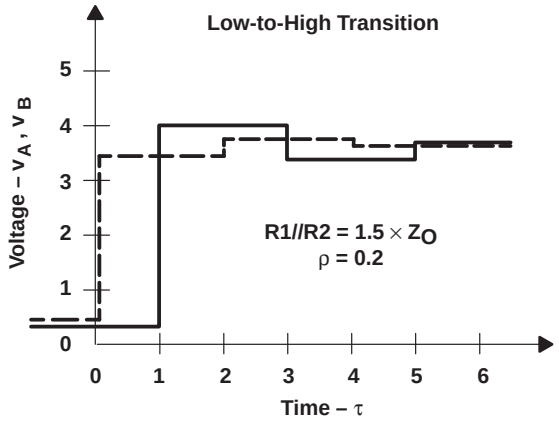

Figure 39. Line Reflections for Mismatch of 50% (ρ = 0.2)

CAUTION:

The supply voltage should be blocked on the terminating network by a ceramic capacitor (C = 0.1 μF) to ensure that the resistors (R1 and R2) cross ac voltage in parallel.

Lines on which there is data transfer in both directions (e.g., bus lines) must be terminated at both ends (see Figure 40). The resistors are dimensioned according to equations 28 and 33. This also ensures that defined high level appears on the bus when all drivers are inactive (3-state).

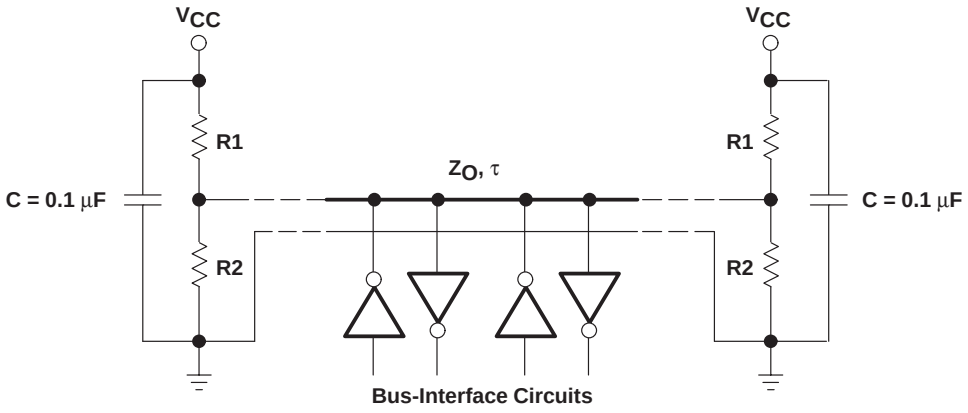

Figure 40. Termination of Bidirectional Bus Lines

5.5 Matching by Series Resistor

As shown in Figure 19, it is also possible to terminate the line at the beginning by appropriate matching of the generator source impedance, which avoids line reflections. In this case, the resistor R_S is connected in series with the output. This is selected so that, together with the source impedance R_O of the gate, it produces the required terminating impedance (see equation 34). The advantage of this circuitry variant is that the power consumption of the system is not increased by the termination.

$$R_O + R_S = Z_O \quad \textbf{(34)}$$

It should be noted that the output impedances of TTL circuits are different for high level (about 30 Ω) and low level (< 10 Ω). Therefore, ideal matching is only possible for one of the two logic levels. Since, as previously mentioned, a certain amount of reflection is admissible, in this case, also, compromises can be made in dimensioning.

The output characteristic is altered by the series resistor R_S (see Figure 41). The fanout of the driver is reduced. In this context note that this kind of line matching is, generally, not applicable to bus lines.

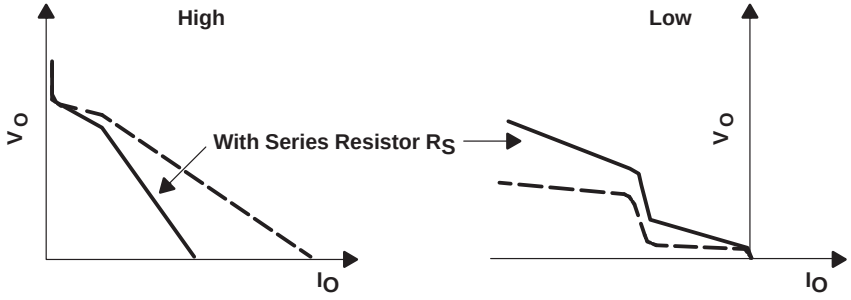

Figure 41. Driver Characteristics for Matching by Series Resistor

Acknowledgment

The author of this document is Karlheinz Fleder.

Appendix

Bergeron Diagrams

This appendix shows the Bergeron diagrams for different kinds of line matching. The output characteristics of the driver and the input characteristics of the receiver correspond to those of the SN74AS00. The following kinds of line termination are discussed:

No Line Termination (Figures 42, 43)

This is the arrangement of most connecting lines on a printed circuit. The positive edge produces a large positive overshoot that is slowly broken down because of the high impedances of the TTL circuits at these voltages. The negative overshoot resulting from the negative edge is largely damped by the clamping diodes at the input of the TTL circuits, meaning that here, also, no negative effects are expected.

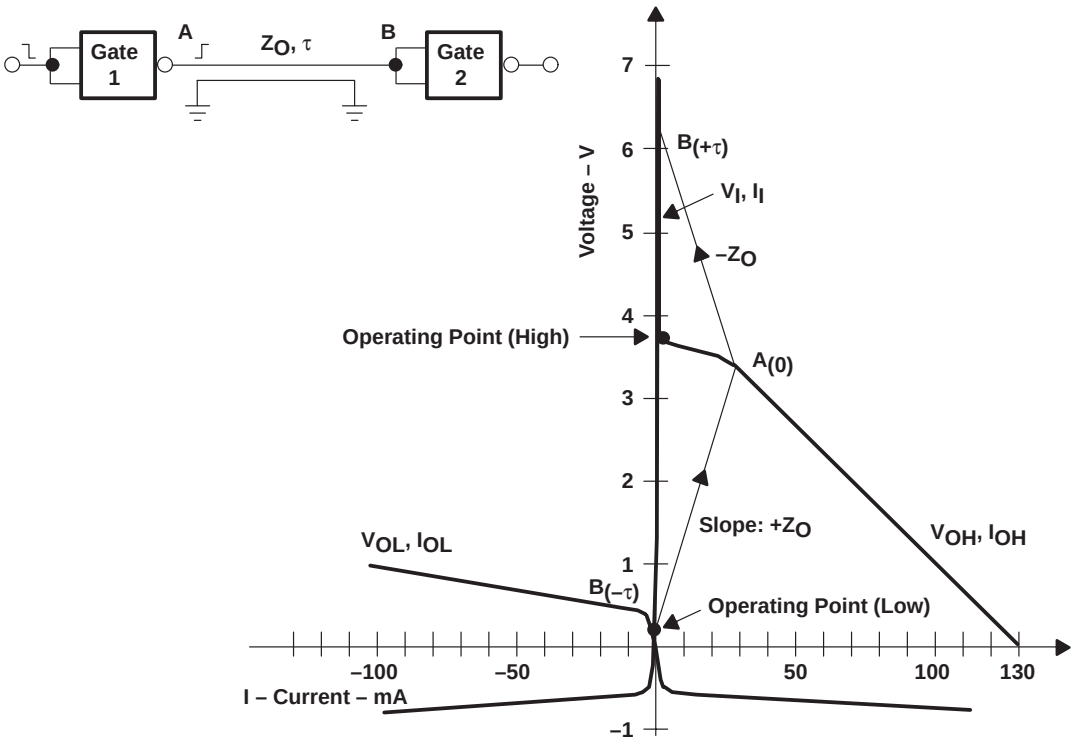

Figure 42. Line Without Termination (Low-to-High Transition, Z_O = 100 Ω)

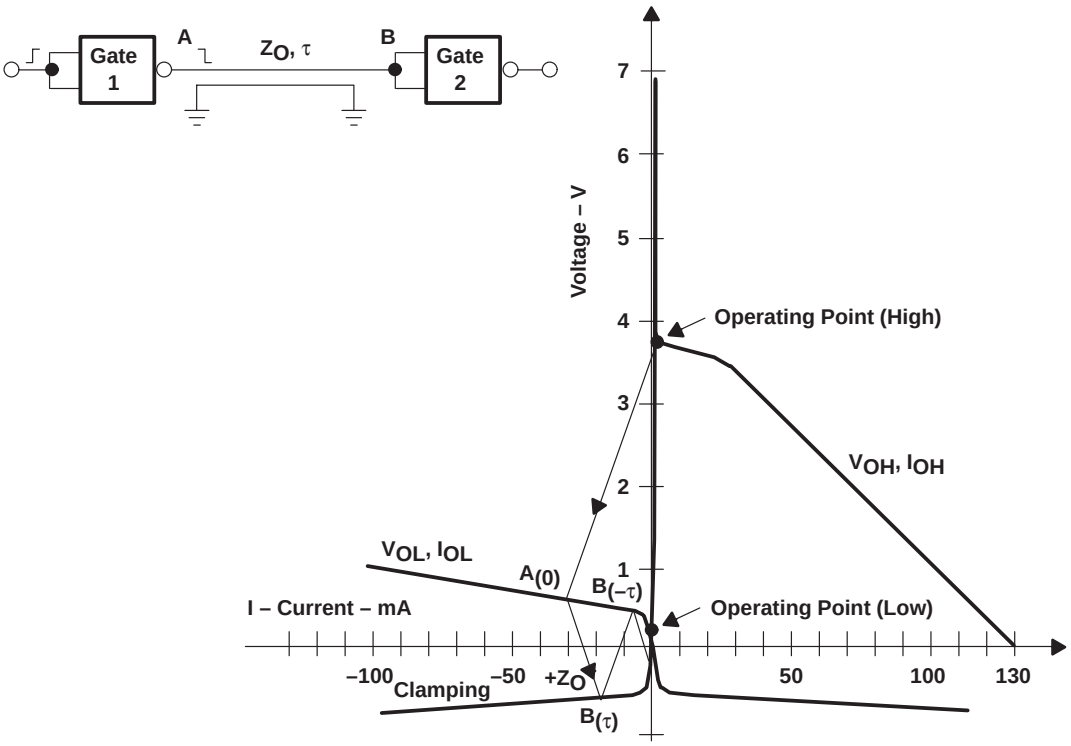

Figure 43. Line Without Termination (High-to-Low Transition, Z_O = 100 Ω)

Line Termination by Resistor to Ground (Figure 44)

In this case, the reflections are entirely suppressed because the line is terminated at the end with its characteristic impedance. However, because of the high power consumption in the termination, this kind of circuitry is not often used.

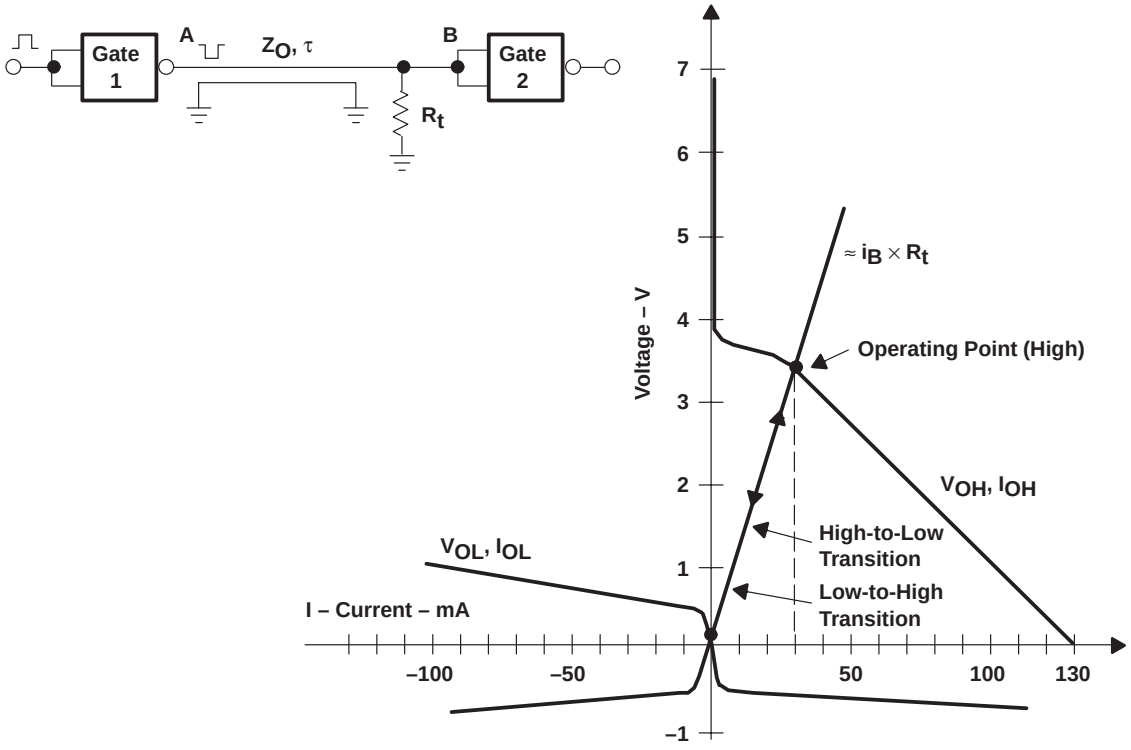

Figure 44. Line Termination by Resistor to Ground, Ideal Matching (R_t = Z_O = 100 Ω)

Line Termination by Series Resistor in the Driver Output (Figure 45)

This kind of line termination requires the least amount of power. However, a disadvantage here is that the voltage at the beginning of the line, after switching, only reaches half amplitude for double the signal-delay time. Consequently, this kind of termination is not suitable for applications where there are several receivers arranged along the line (i.e., bus applications).

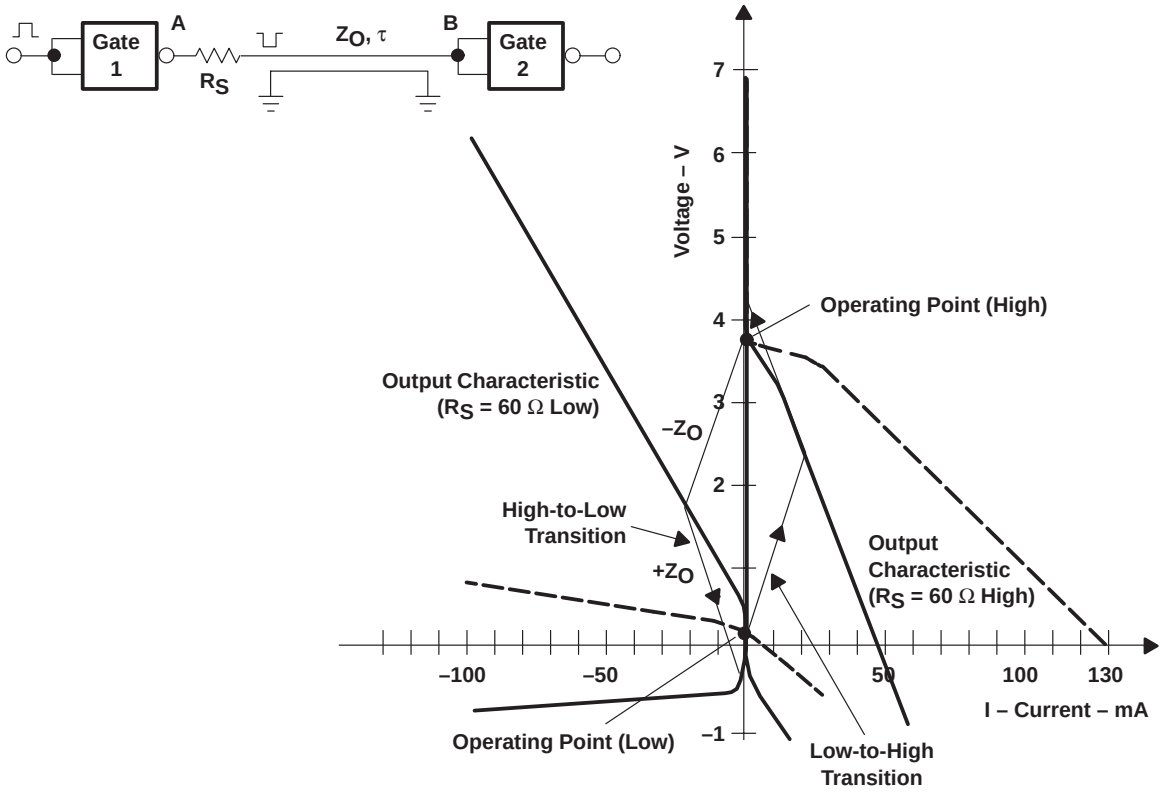

Figure 45. Line Termination by Series Resistor at the Generator Output (Z_O = 100 Ω)

Line Termination by Resistor Network (Split Termination) (Figures 46 and 47)

This is the most common kind of termination for lines that must be terminated. To reduce the power requirements in the terminating network, a certain mismatch will generally be tolerated. As shown in the figures, this does not lead to large signal distortions.

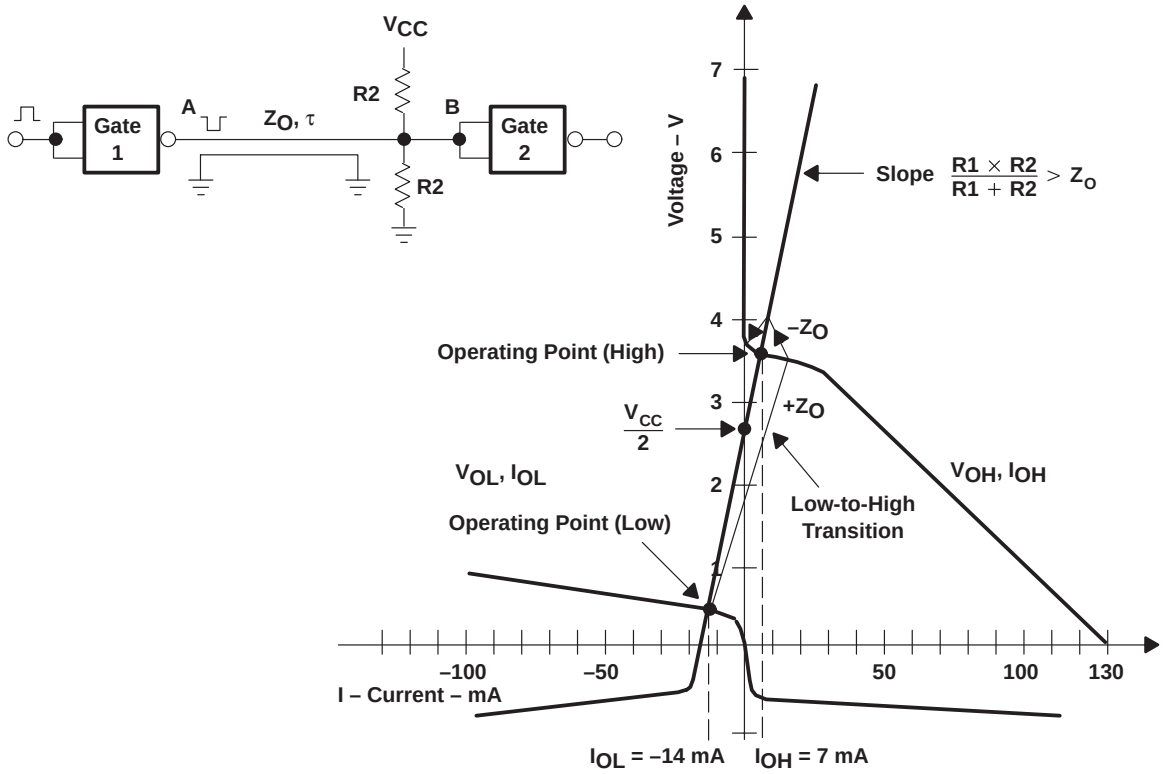

Figure 46. Line Termination by Resistor Network (Low-to-High Transition, Z_O = 100 Ω)

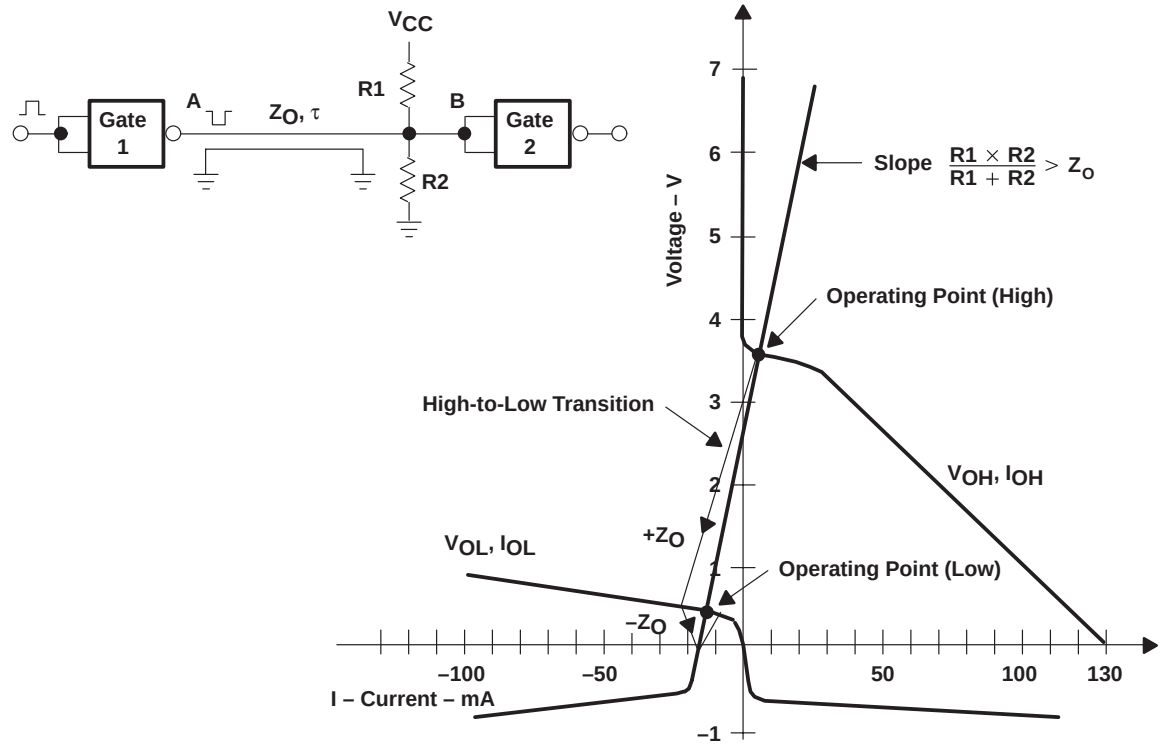

Figure 47. Line Termination by Resistor Network (High-to-Low Transition, Z_O = 100 Ω)