Homework # 2 solutions

17.2 (a) Capacitor C is charged to 10 V and the switch closes at $t = 0$, thus

$$v_O(0+) = 10 \text{ V}$$

Capacitor C then discharges through R exponentially with $v_O(\infty) = 0$

$$v_O(t) = 0 - (0 - 10)\, e^{-t/\tau}$$

$$\Rightarrow v_O(t) = 10e^{-t/\tau}$$

(b) For $C = 100$ pF and $R = 1$ kΩ, we have

$$\tau = 100 \times 10^{-12} \times 1 \times 10^3 = 100 \text{ ns}$$

$$t_{PHL} = 0.69\tau = 0.69 \times 100 = 69 \text{ ns}$$

$$t_f = 2.22\tau = 2.2 \times 100 = 220 \text{ ns}$$

In part b of prob. 17.2, the time for the output to go from 90% to 10% , i.e. fall time, is found. You can find the time it takes to drop from 100% to 10%, then find the time to drop from 100% to 90% and subtract the two. In the solution above they are using 2.2 x tau which is | ln(.1)| - | ln(.9)| and tau is RC

17.4

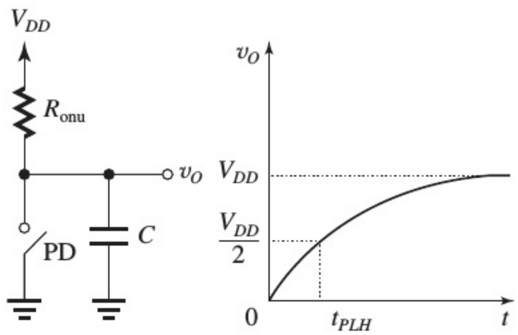

Figure 1(a)

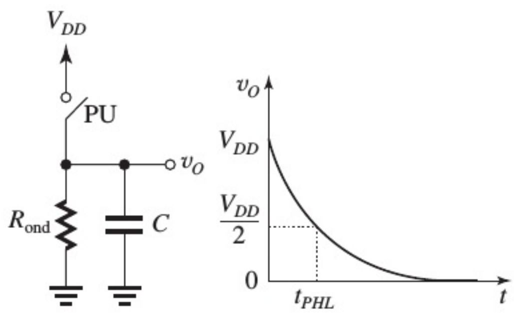

Figure 1(b)

To obtain t_{PLH}, consider the situation in Fig. 1(a). Here, PD has just opened (at $t = 0$), leaving $v_O = 0$ V at $t = 0+$. Capacitor C then charges through the "on" resistance of the pull-up switch, R_{onu}, toward V_{DD}, thus

$$v_O(t) = V_\infty - (V_\infty - V_{0+})e^{-t/\tau}$$

$$= V_{DD} - (V_{DD} - 0)e^{-t/\tau}$$

$$= V_{DD}(1 - e^{-t/\tau})$$

At $t = t_{PLH}$, $v_O = V_{DD}/2$, thus

$$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_{PLH}/\tau})$$

$$\Rightarrow e^{-t_{PLH}/\tau} = 0.5$$

$$\Rightarrow t_{PLH} = \tau \ln 2 = 0.69\tau$$

For $C = 20$ fF, $R_{\text{onu}} = 2$ kΩ, then

$$t_{PLH} = 0.69 \times 20 \times 10^{-15} \times 2 \times 10^3$$

$$= 27.6 \text{ ps}$$

Next we determine t_{PHL} by considering the situation depicted in Fig. 1(b). Here, PU has just opened, leaving $v_O(0+) = V_{DD}$. Capacitor C then discharges through the on resistance of the pull-down switch, R_{ond}, toward 0 V, thus $v_O(\infty) = 0$, thus

$$v_O = 0 - (0 - V_{DD})\, e^{-t/\tau}$$

$$= V_{DD}e^{-t/\tau}$$

At $t = t_{PHL}$, $v_O = V_{DD}/2$ and we get

$$\frac{V_{DD}}{2} = V_{DD}e^{-t_{PHL}/\tau}$$

$$\Rightarrow t_{PHL} = 0.69\tau$$

Here,

$$\tau = CR_{\text{ond}}$$

$$= 20 \times 10^{-15} \times 1 \times 10^3 = 20 \text{ ps}$$

Thus,

$$t_{PHL} = 0.69 \times 20 \simeq 13.8 \text{ ps}$$

The propagation delay t_P can now be obtained as

$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}(27.6 + 13.8) = 20.7 \text{ ps}$$

17.6 (a) $t_P = \frac{1}{2}(t_{PLH} + t_{PHL})$

Since $t_P = 45$ ps, then

$$t_{PLH} + t_{PHL} = 90 \text{ ps} \quad (1)$$

Now, since I_{charge} is half $I_{\text{discharge}}$, then

$$t_{PLH} = 2t_{PHL} \quad (2)$$

Using (1) together with (2) yields

$t_{PLH} = 60$ ps

$t_{PHL} = 30$ ps

(b) Since the propagation delay is directly proportional to C, then the increase in propagation delay by 50%, when the capacitance is increased by 0.1 pF, indicates that the original total capacitance is 0.2 pF.

(c) The reduction of propagation delays by 40% when the load inverter is removed indicates that the load inverter was contributing 40% of the total capacitance found in (b), that is,

$C_{\text{out}} = 0.12$ pF

$C_{\text{load}} = 0.08$ pF