WHEN PLANNING A MEASUREMENT SYSTEM –

MODEL IT & THERE BY :

- DETERMINE SUITABILITY FOR THE DESIRED USE
- ENABLE CORRECT INTEPRETATION OF MEASUREMENTS
- CONFIRM VALIDITY OF DATA
- GUIDE SYSTEM CALIBRATION
  DYNAMIC MEASUREMENTS IN PARTICULAR

GENERAL INPUT-OUTPUT RELATION FOR A LINEAR MEASUREMENT SYSTEM

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots \; a_1 \frac{dy}{dt} + a_0 y = \underbrace{bx}_{F(t)} \qquad (1)$$

x & y are functions of time

n = ORDER OF THE SYSTEM

$n = 0 \Rightarrow$ ZERO ORDER SYSTEM

EQ 1 REDUCES TO $a_0 y = bx$ OR $a_0 y = F(t)$

$$y = Cx = KF(t)$$

Characteristics

- RESPONSE INDEPENDANT OF TIME
- y RESPONDS PROPORTIONAL TO X
- NO ENERGY STORAGE
- THIS IS AN IDEAL RESPONSE
- IN REAL LIFE, CAN ONLY BE APPROXIMATED

$K = \frac{1}{a_0} \equiv$ STATIC SENSITIVITY, i.e. DC GAIN

FIRST ORDER MEASUREMENT SYSTEM

$n = 1$

EQ (1) BECOMES $$a_1 \frac{dy}{dt} + a_0 y = bx \quad = F(t) \qquad (2)$$

REWRITING $$\tau \frac{dy}{dt} + y = kx = kF(t) \qquad (3)$$

WHERE

k = STATIC SENSITIVITY $= \frac{1}{a_0}$

τ = TIME CONSTANT $= \frac{a_1}{a_0}$

EQUATION (3) SOLUTION HAS 2 PARTS

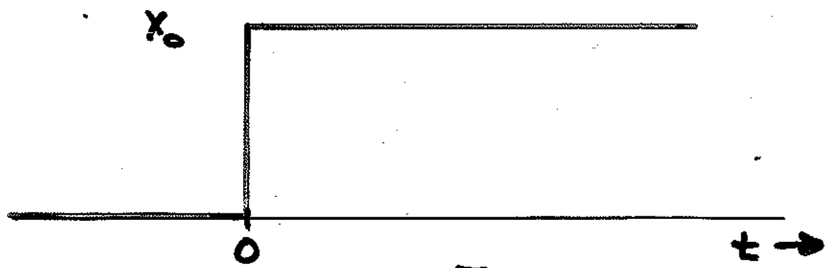
- GENERAL SOLUTION
  - DEPENDS ON SYSTEM ONLY
  - INDEPENDENT OF FORCING FUNCTION x $f(t)$
  - FIND BY SETTING $x = 0$ & SOLVING
  - $y_{gen} = Ce^{-t/\tau}$ OR $(y_0 + C)e^{-t/\tau}$

    $y_0 \Rightarrow$ INITIAL VALUE
- PARTICULAR SOLUTION
  - DEPENDS ON FORCING FUNCTION $x(t)$ $F(t)$

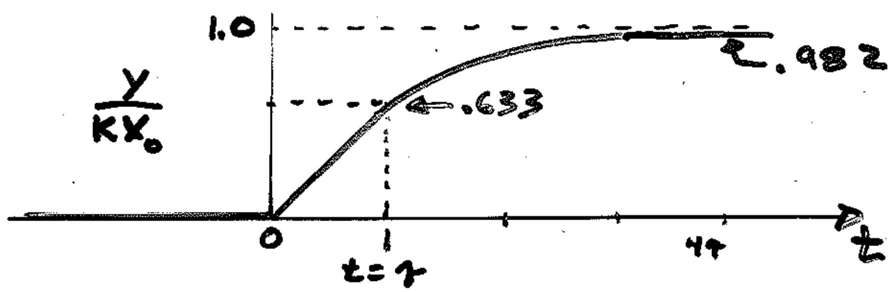
1ST ORDER SYSTEM RESPONSE TO A STEP

LET INPUT & OUTPUT BE ZERO UNTIL $t=0$

AT $t=0$, $x(t)$ RISES TO X_o & REMAINS X_o



INPUT



PARTICULAR SOLUTION ⇒ $y_{PART} = KX_o$

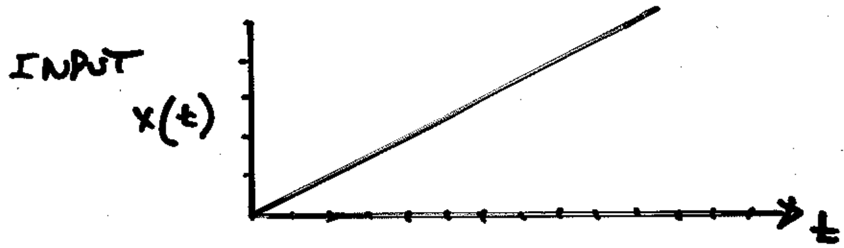
$$y = Ce^{-t/\tau} + kx_o$$

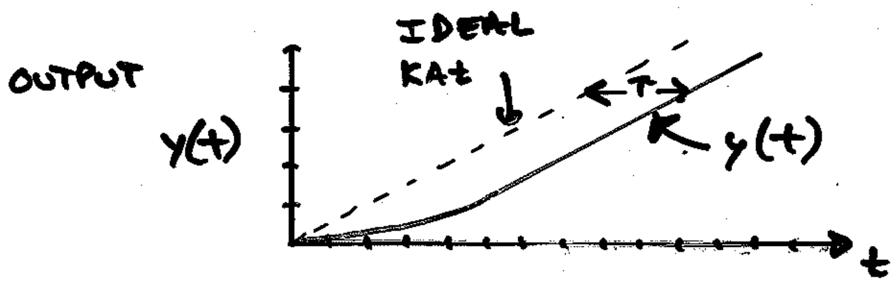
@ $t=0$, $y=0$ $\therefore$ $C = -kx_o$

$$y = -kx_o e^{-t/\tau} + kx_o$$

$$y = \underbrace{Kx_o}_{A}\left(1 - e^{-t/\tau}\right)$$

FIRST ORDER SYSTEM RESPONSE TO A RAMP





FOR $t \le 0$ $y(t) = 0$ & $x(t) = 0$

@ $t > 0$ $x = At$

$$y_{part} = KA(t - \tau)$$

$$y = KA\left(\tau e^{-t/\tau} + t - \tau\right)$$

FIRST ORDER RESPONSE TO A SINE WAVE INPUT

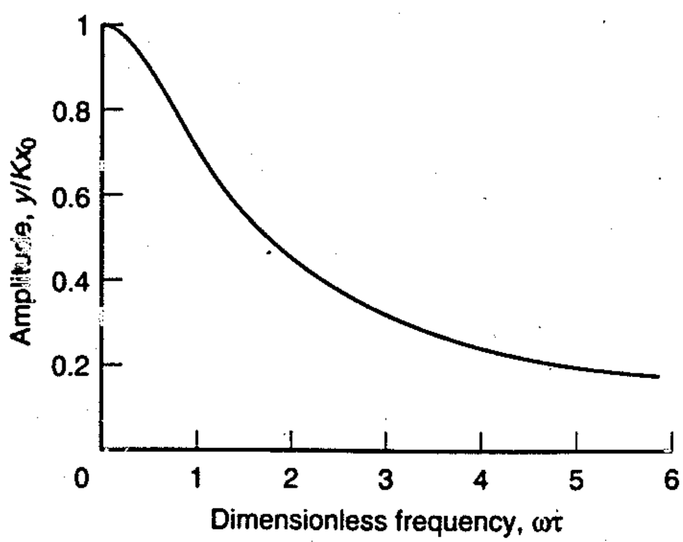
$$x = X_0 \sin \omega t$$

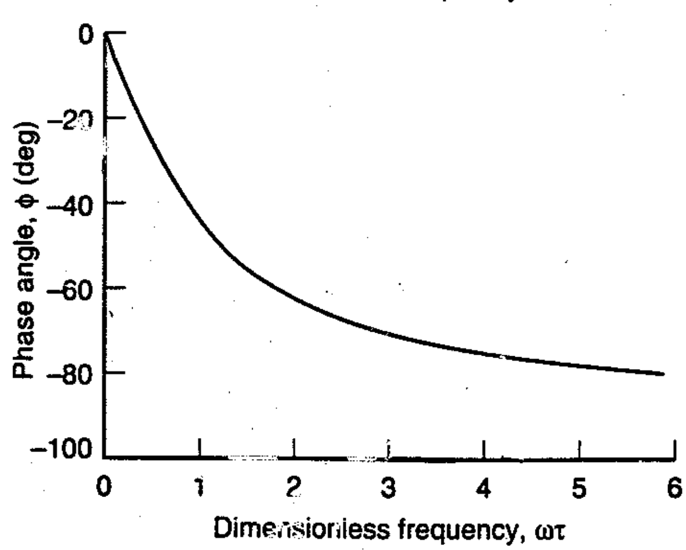
CONTINUING RESPONSE

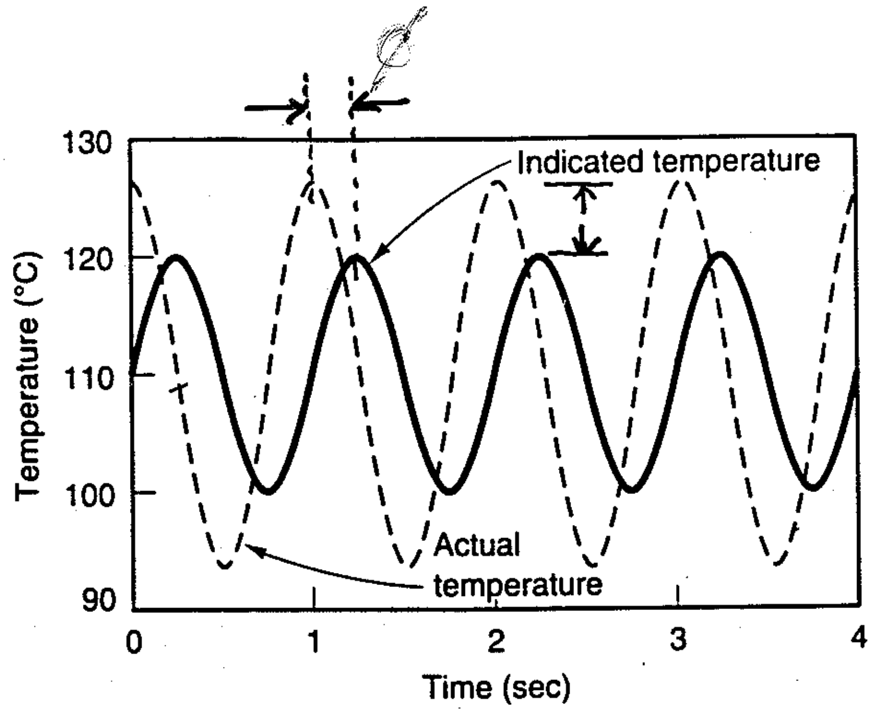
$$y = \frac{K x_0}{\sqrt{1 + \omega^2 \tau^2}} \sin(\omega t + \phi)$$

ϕ = PHASE ANGLE BETWEEN FORCING FUNCT AND OUTPUT RESPONSE

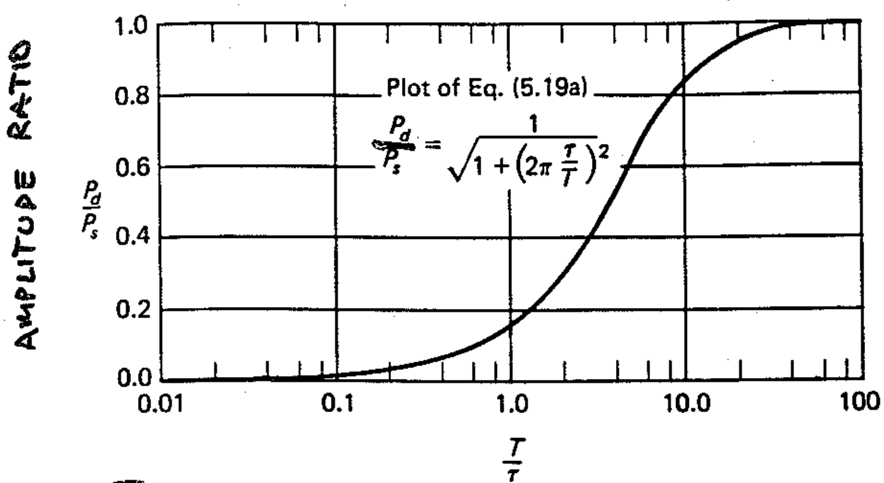
$$\phi = -\tan^{-1} \omega \tau$$





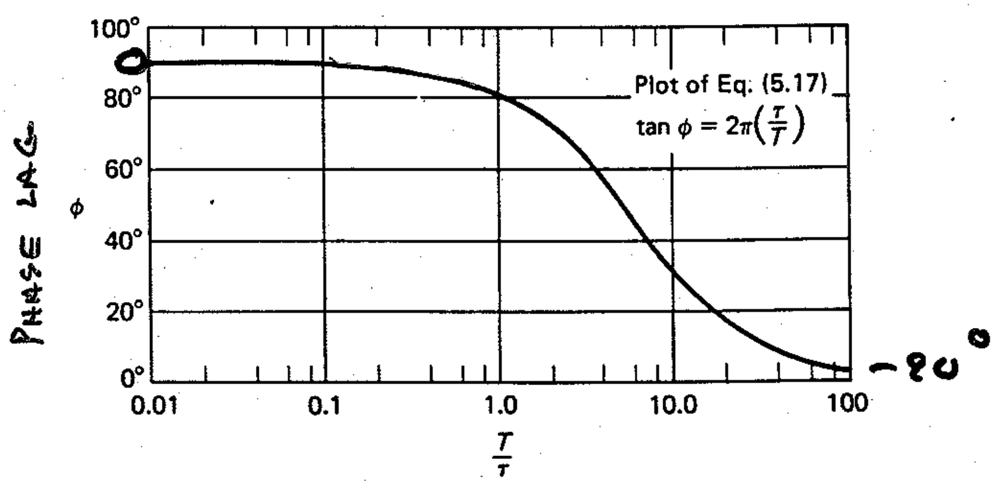

Indicated temperature
Actual
temperature
130
120
110
100
90
Temperature (°C)
0
1
2
3
4
Time (sec)

FIRST ORDER SYSTEM – HARMONICALLY EXCITED



$$\frac{T}{\tau} = \text{RATIO: } \frac{\text{EXCITATION PERIOD OF FORCING FUNCTION}}{\text{SYSTEM TIME CONSTANT}}$$

$$T = \frac{1}{\text{freq hz}} = \frac{2\pi}{\text{freq rad/sec}}$$



Second ORDER SYSTEMS

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b \underset{f(t)}{x}$$

DIVIDE BY a_0 & DEFINE:

$K = \frac{b}{a_0}$ STATIC SENSITIVITY

$\omega_n = \left(\frac{a_0}{a_2}\right)^{\frac{1}{2}}$ UNDAMPED NATURAL FREQ

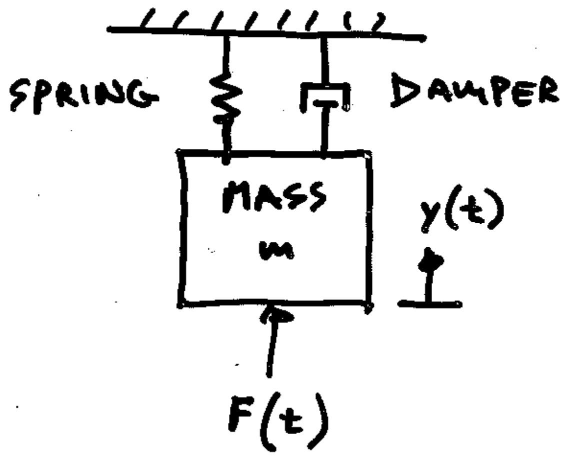
$\delta = \frac{a_1}{2(a_0 a_2)^{\frac{1}{2}}}$ DAMPING RATIO

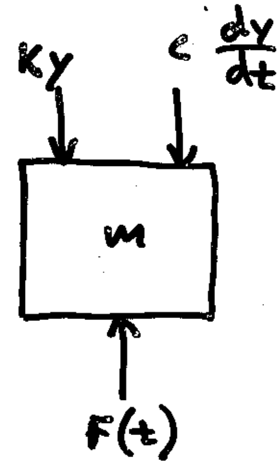
$$\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\delta}{\omega_n}\frac{dy}{dt} + y = K\underset{f(t)}{x}$$

FOR UNDERDAMPED SYSTEMS

RESONANT FREQ $\omega_R = \omega_n\sqrt{1-2\delta^2}$

EXAMPLE SECOND ORDER SYSTEM





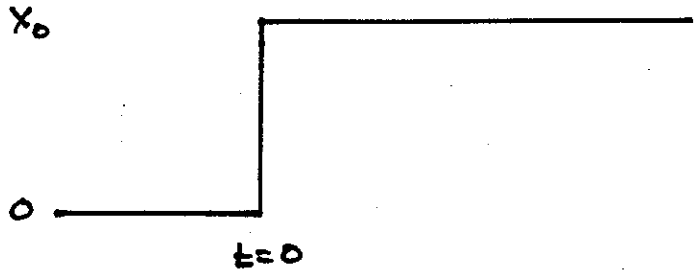
$$\Sigma F = m\left(\frac{d^2y}{dt^2}\right)$$

$$m\frac{d^2y}{dt} + c\frac{dy}{dt} + ky = F(t)$$ y: MASS DISPLACEMENT

$$\left(\frac{1}{\omega_n^2}\frac{d^2y}{dt} + \frac{2\delta}{\omega_n}\frac{dy}{dt} + y = Kx\right)$$ PREVIOUS EQUATION

$$\omega_n = \left(\frac{k}{m}\right)^{\frac{1}{2}} \qquad \delta = \frac{c}{2\sqrt{km}} \qquad K = \frac{1}{k}$$

2ND ORDER SYSTEM – STEP INPUT



$\zeta > 1$:

$$\frac{y}{y_e} = 1 - e^{-\zeta\omega_n t}\left[\cosh(\omega_n t\sqrt{\zeta^2 - 1}) + \frac{\zeta}{\sqrt{\zeta^2 - 1}}\sinh(\omega_n t\sqrt{\zeta^2 - 1})\right]$$

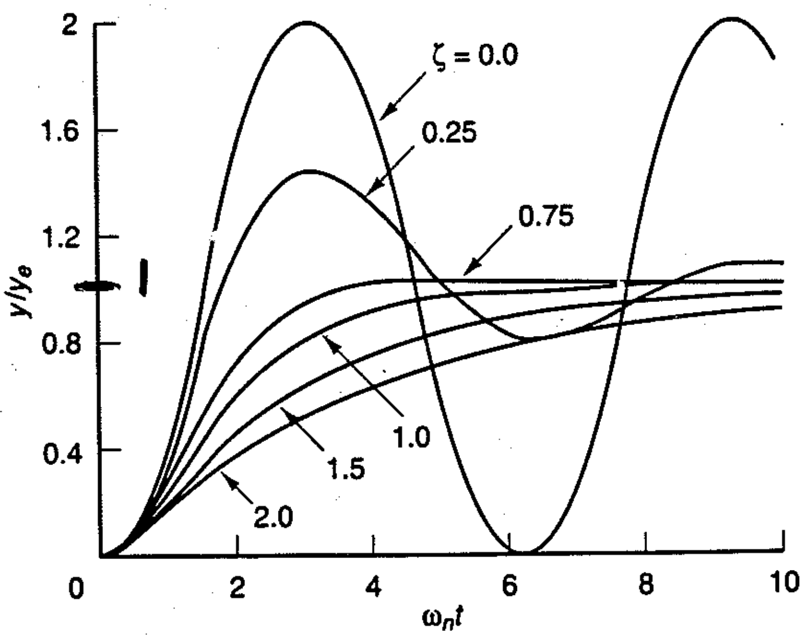
$\zeta = 1$:

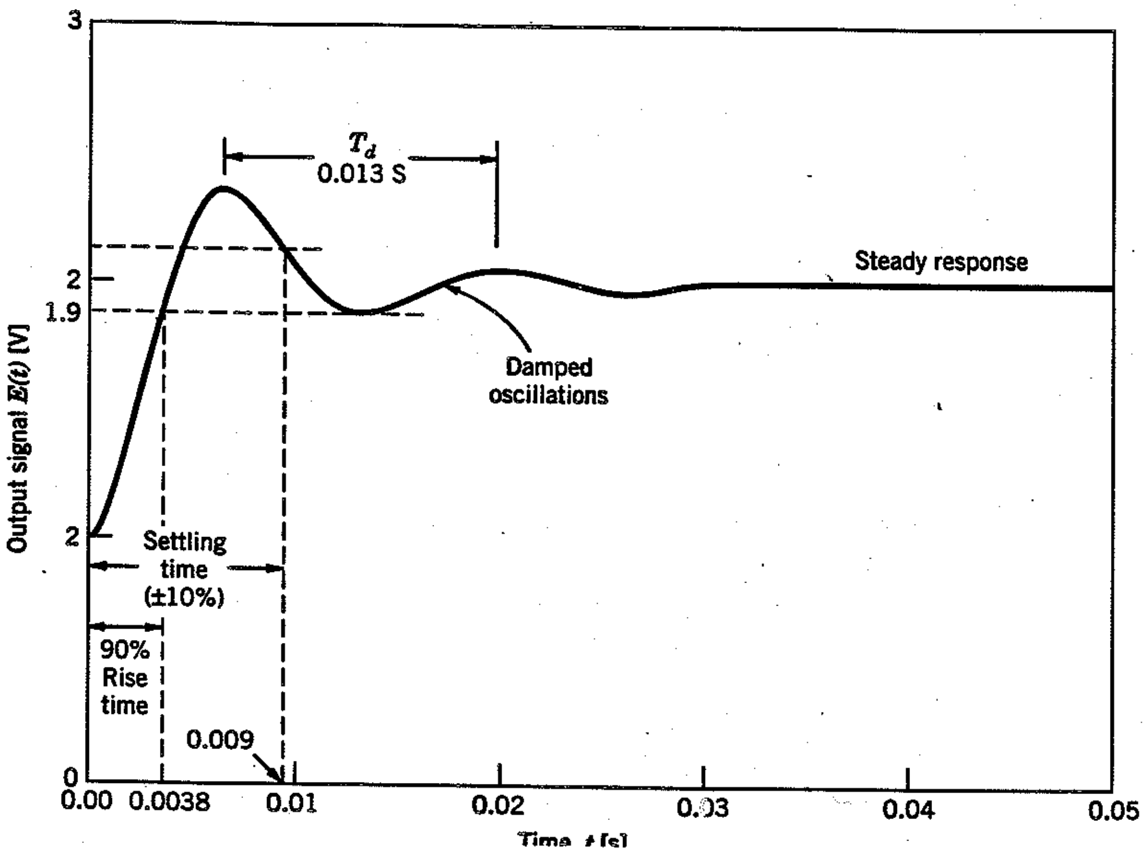
$$\frac{y}{y_e} = 1 - e^{-\omega_n t}(1 + \omega_n t)$$

$\zeta < 1$:

$$[\sin(wt + \phi)]$$

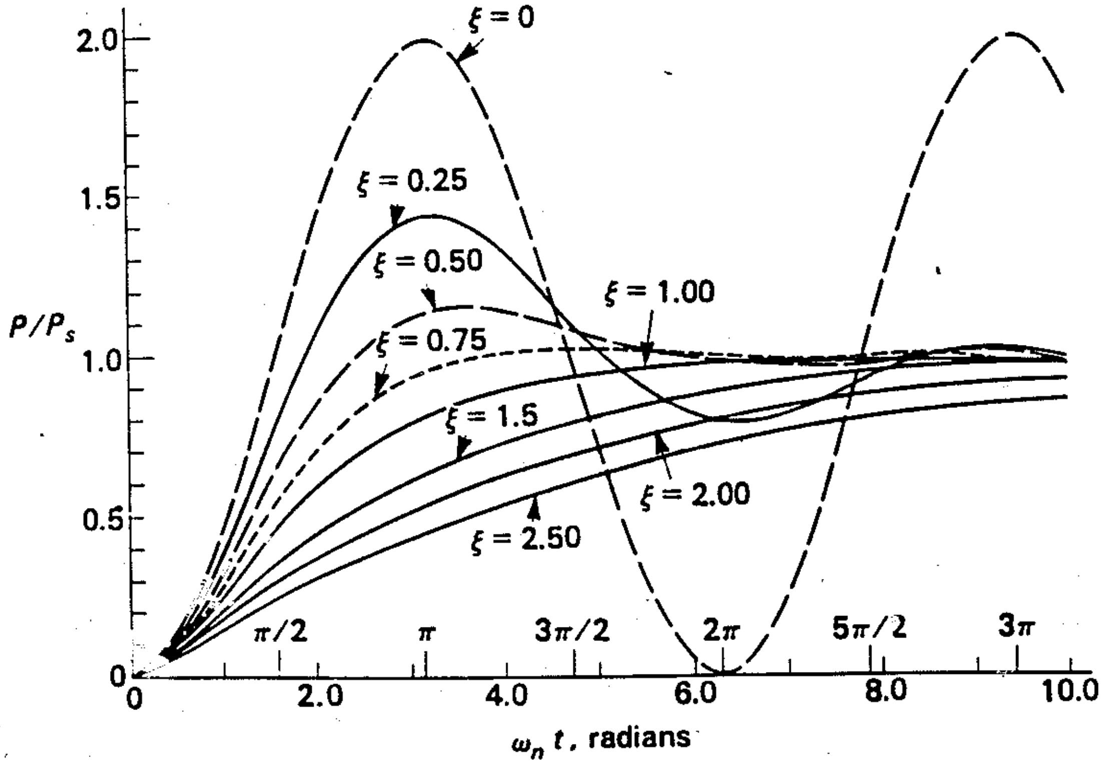
$$\frac{y}{y_e} = 1 - e^{-\zeta\omega_n t}\left[\underbrace{\frac{1}{\sqrt{1-\zeta^2}}}_{1}\sin(\underbrace{\omega_n t\sqrt{1-\zeta^2}}_{W_D = W\sqrt{1-\zeta^2}} + \phi)\right]$$





3
2
1.9
2
0
T_d
0.013 S
Steady response
Damped
oscillations
Settling
time
(±10%)
90%
Rise
time
0.009
0.00
0.0038
0.01
0.02
0.03
0.04
0.05
Output signal E(t) [V]

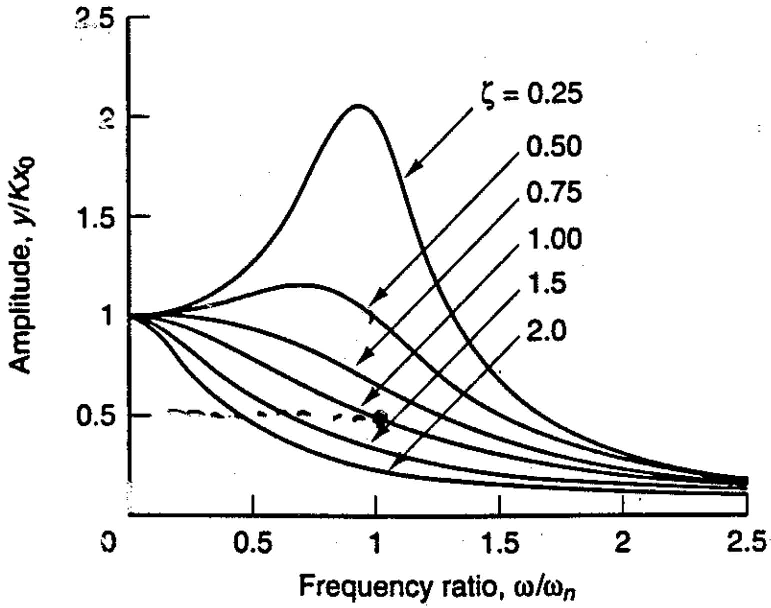
SECOND ORDER SYSTEM
STEP RESPONSE



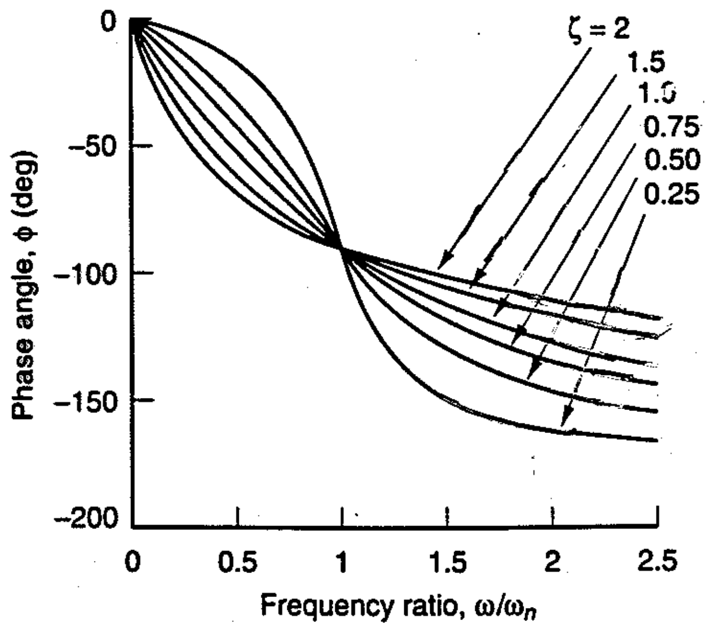
SINE INPUT TO 2ND ORDER

$$\frac{y}{Kx_0} = \frac{1}{[(1 - \omega^2/\omega_n^2)^2 + (2\zeta\omega/\omega_n)^2]^{1/2}} \sin(\omega t + \phi)$$

$$\phi = -\tan^{-1} \frac{2\zeta\omega/\omega_n}{1 - \omega^2/\omega_n^2}$$



(a)



(b)