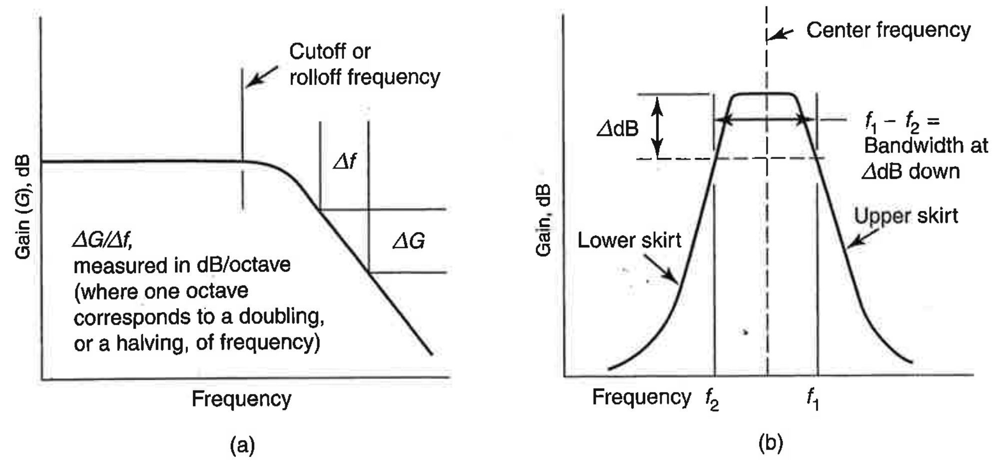

FIGURE 7.24: (a) Some terminology as applied to a low-pass filter; (b) band-pass filter characteristics.

where G is the open-loop gain of the op amp. Because op-amp gains are enormous, the second term in the denominator is usually negligible, and the effective output is just

$$e_o = -\frac{Q(t)}{C_f}$$

Note that the charge amp's output is independent of cable and transducer capacitance [10].

The resistor R_f limits the response of the charge amp at frequencies below $f = 1/2\pi R_f C_f$. Such parallel resistance is often introduced to eliminate low-frequency contributions to output; however, some parallel resistance is always present, owing to the finite resistances of real capacitors.

Although the piezoelectric effect was known in the nineteenth century, it did not become technologically important until very-high-input-impedance amplifiers were developed in the 1950s and 1960s. The charge amp itself was patented by W. P. Kistler in 1950 and gained wide use following the development of MOSFET circuits and high-grade electrical insulators such as Teflon and Kapton [11].

7.16 FILTERS

As we have seen, time-varying measurands commonly consist of a combination of many frequency components or harmonics. In addition, unwanted inputs (noise) are often picked up, thereby resulting in distortion and masking of the true signal. It is usually possible to use appropriate circuitry to selectively filter out some or all of the unwanted noise (see, for example, Section 5.7).

Filtering is the process of attenuating unwanted components of a measurand while permitting the desired components to pass. Filters are of two basic classes, *active* and *passive*. An active filter uses powered components, commonly configurations of op amps, whereas a passive filter is made up of some form of *RLC* arrangement. In addition, filters may be classified by the descriptive terms *high-pass, low-pass, band-pass,* and *notch* or *band-reject*. In each case, reference is to the signal frequency; for example, the high-pass

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filter permits components above a certain cutoff frequency to pass through. The notch filter attenuates a selected band of frequency components, whereas the band-pass filter permits only a range of components about its center frequency to pass. Figures 7.24(a) and (b) illustrate certain terms applied in filter design and use. Similar terms are applicable to the high-pass and notch filters, respectively.

7.17 SOME FILTER THEORY

The simplest low-pass and high-pass filters are made from a single resistor and capacitor. Electrically, these passive *RC* filters are first-order systems (Section 5.18). The *RC* low-pass and high-pass filters are shown in Figs. 7.25(a) and (b), respectively.

Consider first the *RC* low-pass filter. Since a capacitor tends to block low-frequency currents and pass high-frequency currents, the basic effect of the capacitor in this filter is to short-circuit the high-frequency components of the input signal. To determine the frequency characteristics, we must find the filter output, e_o, for a harmonic input voltage, e_i:

$$e_i = V_i \sin(2\pi f t)$$

If negligible current is drawn at the output, the currents through the resistor and the capacitor are equal, so that

$$i = \frac{e_i - e_o}{R} = C\frac{d}{dt}e_o$$

or

$$\frac{d}{dt}e_o + \frac{1}{RC}e_o = \frac{1}{RC}e_i = \frac{V_i}{RC}\sin(2\pi f t) \tag{7.31}$$

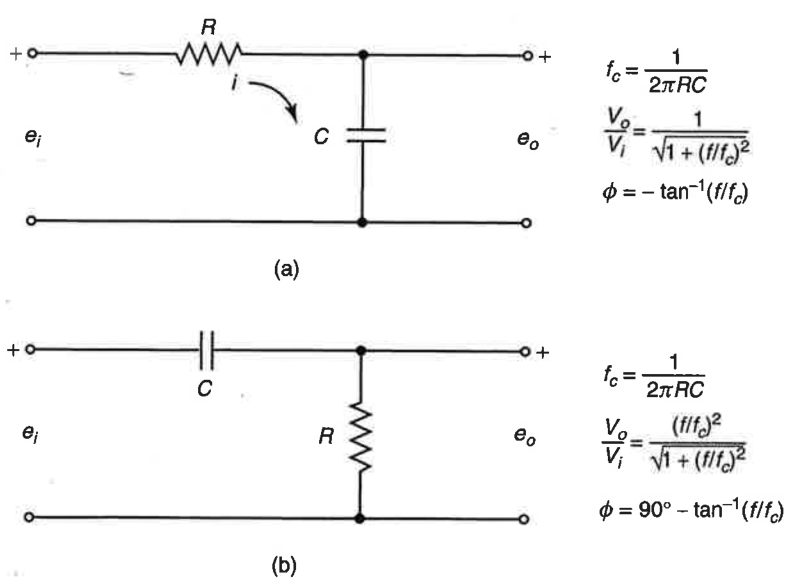


FIGURE 7.25: First-order *RC* filters: (a) low pass, (b) high pass.

Solution of this equation gives (cf. Example 5.7)

$$e_o = V_o \sin(2\pi f t + \phi) = \frac{V_i}{\sqrt{1 + (2\pi RCf)^2}} \sin(2\pi f t + \phi) \tag{7.32}$$

where the phase lag, ϕ, is

$$\phi = -\tan^{-1}(2\pi RCf) \tag{7.32a}$$

Filter performance is normally characterized by defining a *cutoff frequency*; f_c:

$$f_c \equiv \frac{1}{2\pi RC} \tag{7.33}$$

In terms of f_c, the frequency response (or gain), from Eq. (7.32), is

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{1 + (f/f_c)^2}} \tag{7.33a}$$

and the phase response, from Eq. (7.32a), is

$$\phi = -\tan^{-1}\left(\frac{f}{f_c}\right) \tag{7.33b}$$

At the cutoff frequency,

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{P_o}{P_i} = \frac{1}{2} \tag{7.33c}$$

either of which indicates a -3 dB change in the signal strength (see Section 7.12).

The frequency response is shown on linear coordinates in Fig. 7.26. Graphed this way, the filter response seems to change only slightly with frequency. However, the graph

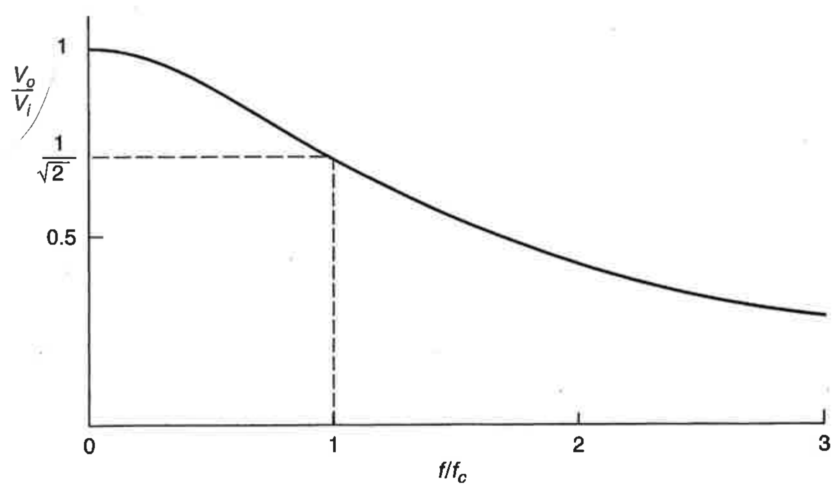


FIGURE 7.26: Frequency response of the *RC* low-pass filter (linear coordinates).

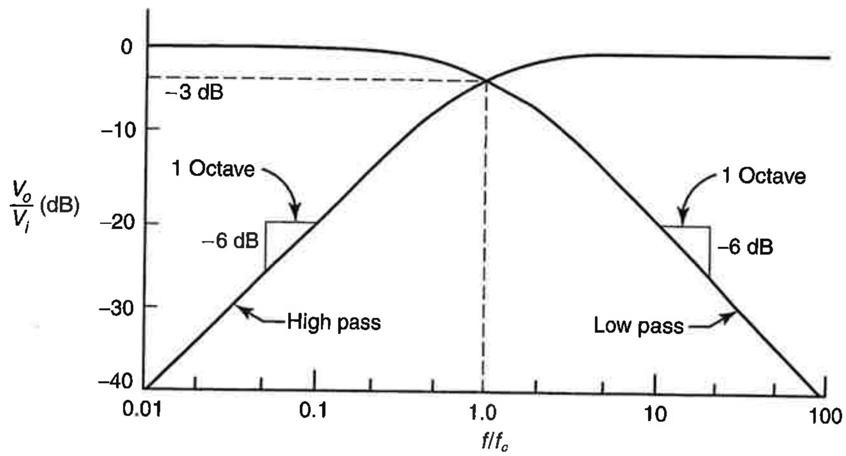


FIGURE 7.27: Frequency response of *RC* low-pass and high-pass filters (Bode plot).

shows only a factor-of-three increase in frequency, while, in practice, filters are used to separate frequencies that may differ by orders of magnitude. A logarithmic graph, such as a Bode plot (Section 7.12), is needed to illustrate such variation.

A Bode plot of the low-pass filter's response is given in Fig. 7.27, illustrating the -3 dB reduction in signal at the cutoff frequency. The frequency range plotted spans four orders of magnitude, and the amplitude attenuation runs from 0 to -40 dB. For frequencies well below f_c, the filter's response is flat and shows no signal reduction. The transition from the passband to rejection band occurs gradually with increasing frequency. In the rejection band itself, at frequencies well above f_c, the amplitude rolloff is -6 dB/octave (an octave being a factor-of-two change in frequency) or -20 dB/decade (a decade being a factor of ten).

In addition to reducing amplitude, this filter also produces an increasing phase shift as signal frequency rises (Fig. 7.28). At the -3 dB point, the output lags the input by 45°.

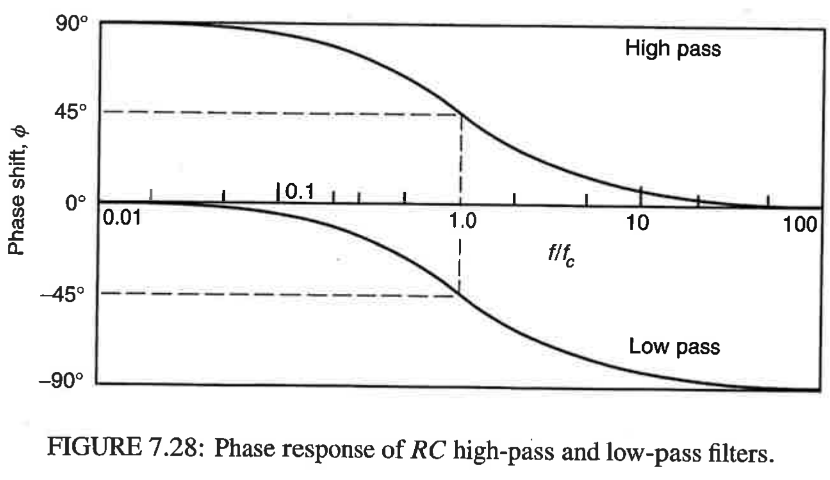


FIGURE 7.28: Phase response of *RC* high-pass and low-pass filters.

The RC high-pass filter is obtained by interchanging the resistor and capacitor [Fig. 7.25(b)]. Now the capacitor blocks low frequencies while passing high frequencies. The results are quite similar:

$$e_o = V_o \sin(2\pi f t + \phi) = \frac{V_i(2\pi RCf)}{\sqrt{1+(2\pi RCf)^2}} \sin(2\pi f t + \phi)$$

where the phase shift, ϕ, is now a *lead* ($\phi > 0$) rather than a lag ($\phi < 0$)

$$\phi = 90^\circ - \tan^{-1}(2\pi RCf)$$

The high-pass filter's cutoff frequency is identical to the low-pass filter's:

$$f_c \equiv \frac{1}{2\pi RC} \tag{7.34}$$

In terms of the cutoff frequency, the frequency response and phase lead are

$$\frac{V_o}{V_i} = \frac{(f/f_c)}{\sqrt{1+(f/f_c)^2}}, \tag{7.34a}$$

$$\phi = 90^\circ - \tan^{-1}\left(\frac{f}{f_c}\right) \tag{7.34b}$$

The -3 dB point is again f_c, and the rolloff in the rejection band is again -6 dB/octave or -20 dB/decade. The high-pass frequency and phase response are shown in Figs. 7.27 and 7.28. One common use of this filter is to remove dc ($f = 0$) offsets.

First-order RC filters have a fairly slow rolloff above the cutoff frequency (not many decibels per octave), but their simplicity still gains them wide use in situations where the desired and undesired frequencies are widely separated. Similar high-pass and low-pass filters can be made using a single resistor–inductor pair. However, first-order RL filters are seldom used.

EXAMPLE 7.10

A transducer responding to a 5000-Hz signal also picks up 60 Hz noise. The resulting output is

$$\{5\sin(2\pi \cdot 60 \cdot t) + 25\cos(2\pi \cdot 5000 \cdot t)\} \text{ mV}$$

To remove the 60-cycle noise, a high-pass filter with cutoff of 1000 Hz is introduced. What is the filtered output?

Solution The amplitude and phase shift are computed separately for each component:

$$\left(\frac{V_o}{V_i}\right)_{60} = \frac{(60/1000)}{\sqrt{1+(60/1000)^2}} = 0.060,$$

$$\left(\frac{V_o}{V_i}\right)_{5000} = \frac{(5000/1000)}{\sqrt{1+(5000/1000)^2}} = 0.98,$$

$$\phi_{60} = 90^\circ - \tan^{-1}\left(\frac{60}{1000}\right) = 86.6^\circ = 1.51 \text{ rad},$$

$$\phi_{5000} = 90^\circ - \tan^{-1}\left(\frac{5000}{1000}\right) = 11.3^\circ = 0.197 \text{ rad}$$

Then

$$e_0 = \{0.3\sin(2\pi \cdot 60 \cdot t + 1.51) + 24.5\cos(2\pi \cdot 5000 \cdot t + 0.197)\}\ \text{mV}$$

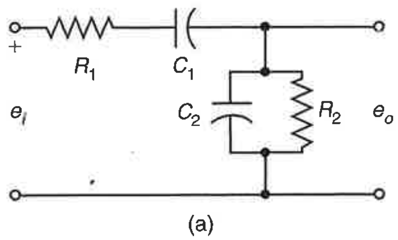
The noise amplitude is reduced from 20% of the signal amplitude to only 1.2%. Note that the signal itself undergoes a slight amplitude reduction as well as a small phase shift. Such changes in the signal are undesirable, and they often motivate the use of more complex filters.

Three desirable elements of filter performance are as follows:

1. Nearly flat response over the pass and rejection bands;
2. High values of rolloff for low-and high-pass filters, as measured in decibels per octave;
3. Steep skirts for band-pass and band-rejection filters.

Significant improvements in performance are obtained by using combinations of several capacitors, inductors, or resistors to produce second-order (or higher-order) electrical response. Such filters can have steeper rolloff and sharper transition from pass to rejection bands. In addition, such compound *RLC* arrangements can produce band-pass and notch filters. For example, Figs. 7.29(a) and (b) show an *RC* band-pass filter and its response:

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{[1 + R_1/R_2 + C_2/C_1]^2 + [2\pi R_1 C_2 f - (1/2\pi R_2 C_1 f)]^2}} \tag{7.35}$$



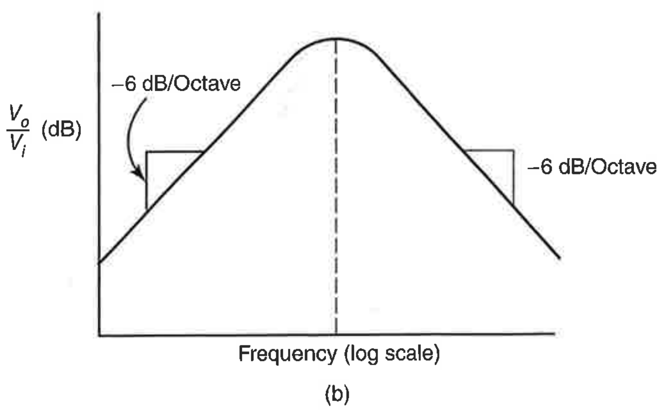


FIGURE 7.29: (a) A circuit for a simple band-pass filter; (b) performance characteristics of band-pass filter shown in (a).

Inductors and capacitors used together allow resonant behavior, which can produce steeper filter skirts than are possible with first-order *RC* circuits. In fact, the resonant circuit of Section 7.11 is sometimes used to build very narrow band-pass filters known as *tuned filters*. Some additional *LC* designs are shown in Fig. 7.30.

Two practical issues influence the design and use of passive filters. First, the filters considered here are all designed as if negligible current is drawn from the output terminals. If several filters are placed in series, to steepen rolloff, then the current drawn by one filter

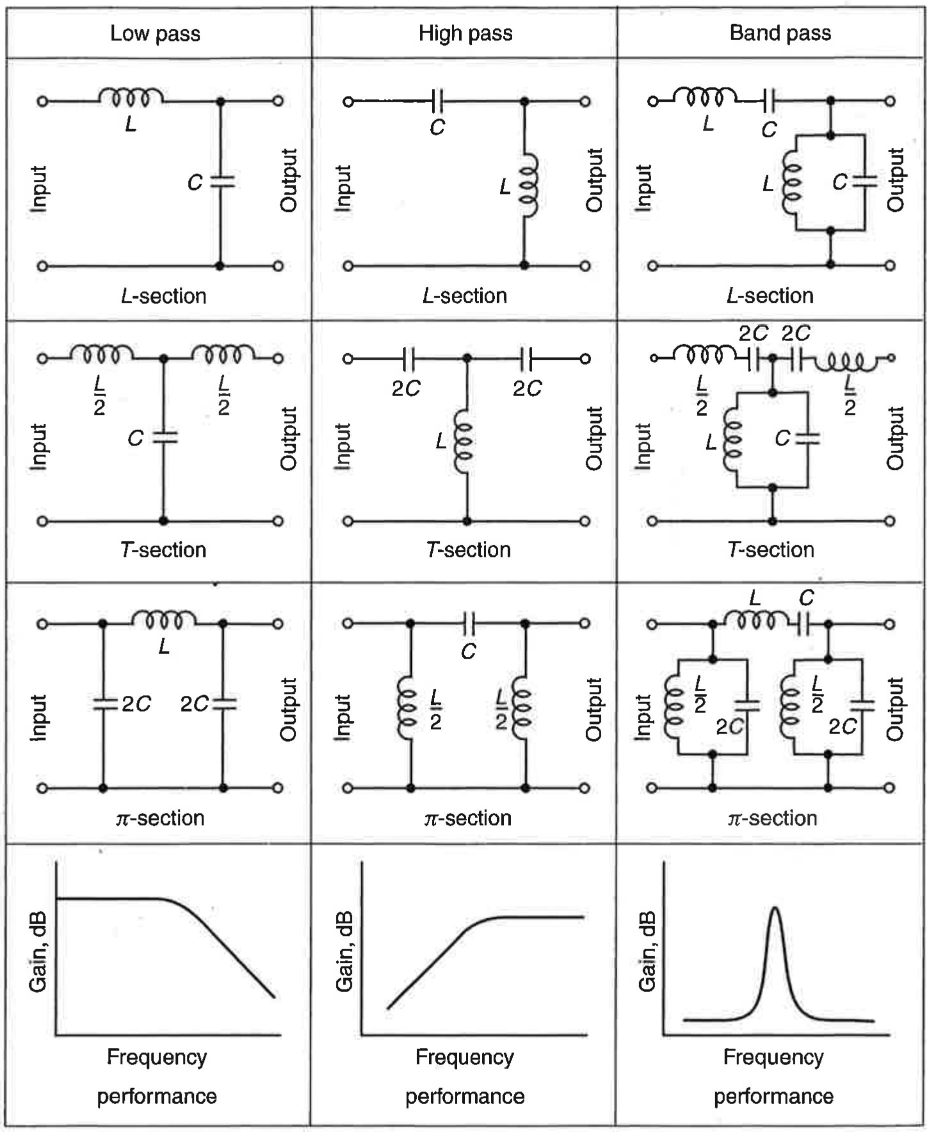


FIGURE 7.30: Examples of *LC* filter arrangements and their output characteristics.

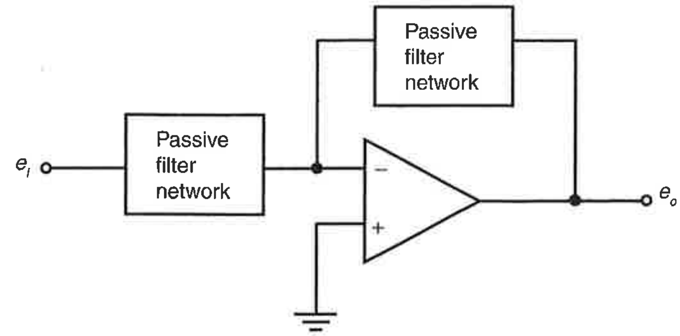


FIGURE 7.31: Basic active filter circuit.

can alter the performance of the filter that precedes it. To avoid this output loading, a voltage follower (Example 7.3) should be introduced as a buffer between each successive filter.

The inductors themselves are the second problem. At the frequencies encountered in mechanical measurements, which rarely exceed 100 kHz, the required inductors may be quite large and bulky. In addition, the optimal inductor values are not always easily obtained, and the inductors may have substantial internal resistances as well. As often happens in engineering, inductors can be much less satisfactory in practice than they seem on paper. The usual way of avoiding these problems is to employ an active filter, as described next.

7.18 ACTIVE FILTERS

Op amps can be used to construct filter circuits without inductors and without the problems of output loading. These active filters can also have very steep rolloff, arbitrarily flat passbands, and even adjustable cutoff frequencies. Active filters are a rich subject, and entire textbooks have been devoted to their design.

The basic active filter is shown in Fig. 7.31. Passive filter networks are linked to an op amp, which provides power and improves impedance characteristics. The passive network is built from resistors and capacitors only: Inductive characteristics are simply simulated by the circuit. Since the output impedance is generally low, these filters can deliver an output current without reduced performance. Some typical active filters are shown in Fig. 7.32.

Active filters are available with rolloffs of 80 dB/octave and more than 60 dB attenuation in the rejection band. High-order active filters are even sold as integrated circuits contained in a single DIP package. For further study, see the Suggested Readings at the end of this chapter.

7.19 DIFFERENTIATORS AND INTEGRATORS

A final op-amp application is in circuits that respond to the rate of change or the time history of an input signal, called *differentiators* and *integrators*, respectively [Figs. 7.33(a) and (b)].

In the differentiator, the currents through the resistor and capacitor are equal, and $e_- = e_+ = 0$. Thus

$$C\frac{d}{dt}e_i = -\frac{e_o}{R}$$