9.5 CONTINUOUS CONTROLLER MODES

The most common controller action used in process control is one or a combination of continuous controller modes. In these modes, the output of the controller changes smoothly in response to the error or rate of change of error. These modes are an extension of the discontinuous types discussed in the previous section.

9.5.1 Proportional Control Mode

The two-position mode had the controller output of either 100% or 0%, depending on the error being greater or less than the neutral zone. In multiple-step modes, more divisions of controller outputs versus error are developed. The natural extension of this concept is the *proportional mode,* where a smooth, linear relationship exists between the controller output and the error. Thus, over some range of errors about the setpoint, each value of error has a unique value of controller output in one-to-one correspondence. The range of error to cover the 0% to 100% controller output is called the *proportional band,* because the one-to-one correspondence exists only for errors in this range. This mode can be expressed by

$$p = K_P e_p + p_0 \tag{9.14}$$

where K_P = proportional gain between error and controller output (% per %)
p_0 = controller output with no error (%)

Direct and Reverse Action Recall that the error in Equation (9.14) is expressed using the difference between setpoint and the measurement, $r - b$. This means that as the measured value increases above the setpoint, the error will be negative and the output will decrease. That is, the term $K_P\, e_p$ will subtract from p_0. Thus, Equation (9.14) represents reverse action. Direct action would be provided by putting a negative sign in front of the correction term.

A plot of the proportional mode output versus error for Equation (9.14) is shown in Figure 9.11. In this case, p_0 has been set to 50% and two different gains have been used. Note that the proportional band is dependent on the gain. A high gain means large response to an error, but also a narrow error band within which the output is not saturated.

In general, the proportional band is defined by the equation

$$PB = \frac{100}{K_P} \tag{9.15}$$

Let us summarize the characteristics of the proportional mode and Equation (9.14).

1. If the error is zero, the output is a constant equal to p_0.
2. If there is error, for every 1% of error, a correction of K_P percent is added to or subtracted from p_0, depending on the sign of the error.
3. There is a band of error about zero of magnitude PB within which the output is not saturated at 0% or 100%.

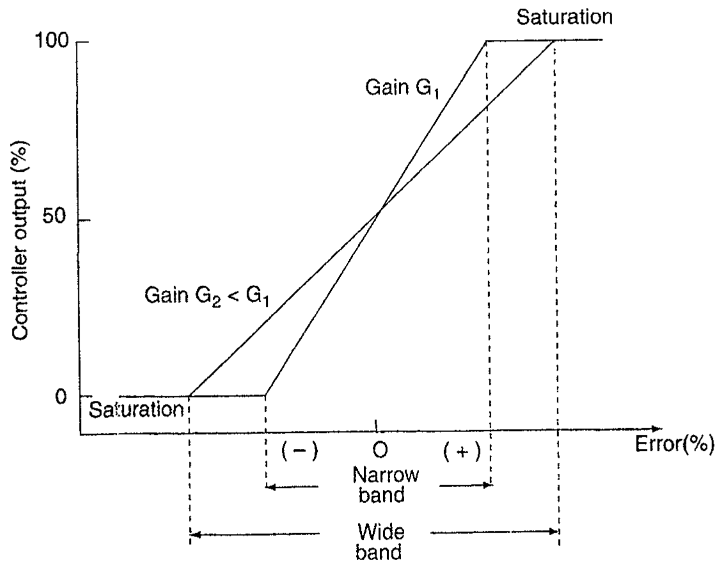

FIGURE 9.11
The proportional band of a proportional controller depends on the inverse of the gain.

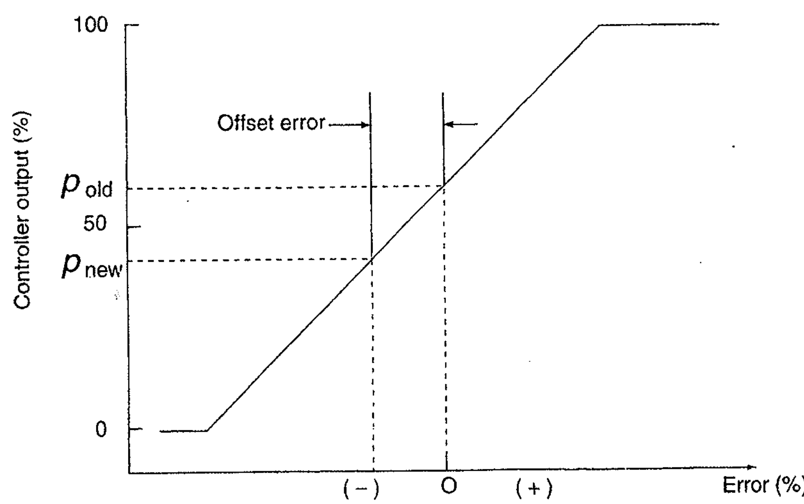

FIGURE 9.12
An offset error must occur if a proportional controller requires a new zero-error output following a load change.

Offset An important characteristic of the proportional control mode is that it produces a permanent *residual error* in the operating point of the controlled variable when a change in load occurs. This error is referred to as *offset.* It can be minimized by a larger constant, K_P, which also reduces the proportional band. To see how offset occurs, consider a system under nominal load with the controller at 50% and the error zero, as shown in Figure 9.12.

If a transient error occurs, the system responds by changing controller output in correspondence with the transient to effect a return-to-zero error. Suppose, however, a load change occurs that requires a permanent change in controller output to produce the zero-error state. Because a one-to-one correspondence exists between controller output and error, it is clear that a new, zero-error controller output can *never* be achieved. Instead, the system produces a small permanent offset in reaching a compromise position of controller output under new loads.

EXAMPLE 9.6

Consider the proportional-mode level-control system of Figure 9.13. Value A is linear, with a flow scale factor of 10 m^3/h per percent controller output. The controller output is nominally 50% with a constant of $K_P = 10\%$ per %. A load change occurs when flow through valve B changes from 500 m^3/h to 600 m^3/h. Calculate the new controller output and offset error.

Solution

Certainly, valve A must move to a new position of 600 m^3/h flow or the tank will empty. This can be accomplished by a 60% new controller output because

$$Q_A = \left(\frac{10\ \text{m}^3/\text{h}}{\%}\right)(60\%) = 600\ \text{m}^3/\text{h}$$

as required. Because this is a proportional controller, we have

$$p = K_P e_p + p_0$$

with the nominal condition $p_0 = 50\%$. Thus

$$e_p = \frac{p - p_0}{K_P} = \frac{60 - 50}{10}\%$$

$$e_p = 1\%$$

so a 1% offset error occurred because of the load change.

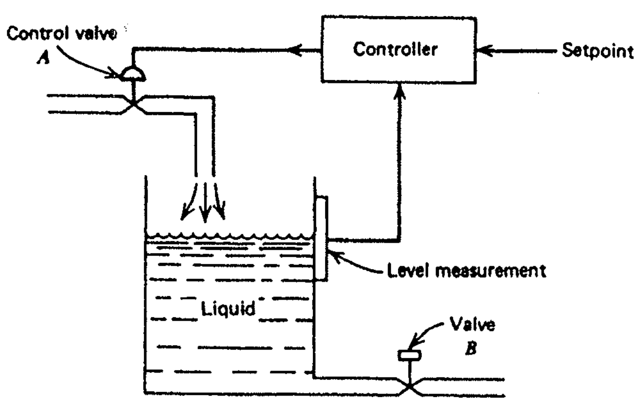

FIGURE 9.13
Level-control system for Example 9.6.

Application The offset error limits use of the proportional mode to only a few cases, particularly those where a manual reset of the operating point is possible to eliminate offset. Proportional control generally is used in processes where large load changes are unlikely or with moderate to small process lag times. Thus, if the process lag time is small, the proportional band can be made very small (large K_P), which reduces offset error. Figure 9.11 shows that if K_P is made very large, the PB becomes very small, and the proportional mode acts just like an ON/OFF mode. Remember that the ON/OFF mode exhibited oscillations about the setpoint. From these statements it is clear that, for high gain, the proportional mode causes oscillations of the error.

9.5.2 Integral-Control Mode

The offset error of the proportional mode occurs because the controller cannot adapt to changing external conditions—that is, changing loads. In other words, the zero-error output is a fixed value. The integral mode eliminates this problem by allowing the controller to adapt to changing external conditions by changing the zero-error output.

The need for integral action shows up when it is noted that even with proportional action correction, the error does not go to zero in time. Suppose a system has some error, e_p, and the proportional mode provides a change in controller output, $K_P e_p$. As we watch the error in time, we note that the error may reduce, but *it does not go to zero;* in fact, it may become constant. Integral action is needed.

Integral action is provided by summing the error over time, multiplying that sum by a gain, and adding the result to the present controller output. You can see that if the error makes random excursions above and below zero, the net sum will be zero, so the integral action will not contribute. But if the error becomes positive or negative for an extended period of time, the integral action will begin to accumulate and make changes to the controller output.

In the mathematics of continuous functions, such as error, summation is represented by integration. Therefore, this mode is represented by an integral equation

$$p(t) = K_I \int_0^t e_P \, dt + p(0) \tag{9.16}$$

where $p(0)$ is the controller output when the integral action starts. The gain K_I expresses how much controller output in percent is needed for every percent-time accumulation of error.

Another way of thinking of integral action is found by taking the derivative of Equation (9.16). In that case, we find a relation for the rate at which the controller output changes,

$$\frac{dp}{dt} = K_I e_p \tag{9.17}$$

This equation shows that when an error occurs, the controller begins to increase (or decrease) its output at a rate that depends upon the size of the error and the gain. If the error is zero, the controller output is not changed. If there is positive error, the controller output begins to ramp up at a rate determined by Equation (9.17). Figure 9.14 illustrates this for two different values of gain.

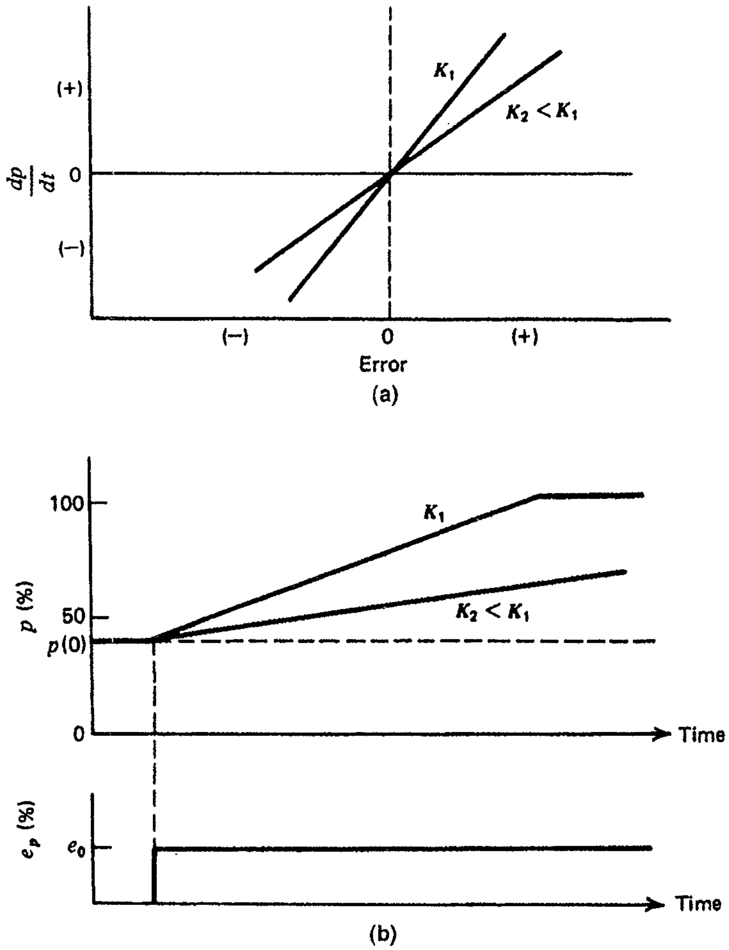

FIGURE 9.14
Integral mode controller action: (a) The rate of output change depends on error, and (b) an illustration of integral mode output and error.

Figure 9.14a shows how the *rate of change* of controller output depends upon the value of error and the size of the gain. Figure 9.14b shows how the actual controller output would look if a constant error occurred. You can see how the controller output begins to ramp up at a rate determined by the gain. In the case of gain K_1, the output finally saturates at 100%, and no further action can occur (perhaps a control valve is fully open, for example).

Let us summarize the characteristics of the integral mode and Equation (9.16).

1. If the error is zero, the output stays fixed at a value equal to what it was when the error went to zero.
2. If the error is not zero, the output will begin to increase or decrease at a rate of K_I percent/second for every 1% of error.

Area Accumulation From calculus we learn that an integral determines the area of the function being integrated. Thus, Equation (9.16) can be interpreted as providing a controller output equal to the net area under the *error-time curve* multiplied by K_I. We often say that the integral term *accumulates* error as a function of time. Thus, for every $1\% - s$ of accumulated error-time area, the output will be K_I percent.

EXAMPLE 9.7 An integral controller is used for speed control with a setpoint of 12 rpm within a range of 10 to 15 rpm. The controller output is 22% initially. The constant $K_I = -0.15\%$ controller output per second per percentage error. If the speed jumps to 13.5 rpm, calculate the controller output after 2 s for a constant e_p.

Solution

We find e_p from Equation (9.3):

$$e_p = \frac{r - b}{b_{max} - b_{min}} \times 100$$

$$e_p = \frac{12 - 13.5}{15 - 10} \times 100$$

$$e_p = -30\%$$

The rate of controller output change is then given by Equation (9.17),

$$\frac{dp}{dt} = K_I e_p = (-0.15\ s^{-1})(-30\%)$$

$$\frac{dp}{dt} = 4.5\%/s$$

The controller output for constant error will be found from Equation (9.16)

$$p = K_I \int_0^t e_p\, dt + p(0)$$

but because e_p is constant,

$$p = K_I e_p t + p(0)$$

After 2 s, we have

$$p = (0.15)(30\%)(2) + 22$$

$$p = \mathbf{31\%}$$

The integral gain, K_I, is often represented by the inverse, which is called the *integral time*, or the *reset action*, $T_I = 1/K_I$. This is often expressed in minutes instead of seconds because this unit is more typical of many industrial process speeds.

The integral controller constant K_I may be expressed in percentage change per *minute* per percentage error, whenever a typical process-control loop has characteristic response time in minutes rather than seconds. Thus, an integral mode controller with reset action at 5.7 minutes means that K_I for our equations would be

$$K_I = \frac{1}{(5.7\ \text{min})(60\ \text{s/min})}$$

$$K_I = 2.92 \times 10^{-3}\ \text{s}^{-1}$$

Applications Use of the integral mode is shown by the flow control system in Figure 9.9, except that we now assume a reverse-acting integral controller mode. Operation can be understood using Figure 9.15. A load change-induced error occurs at $t = 0$.

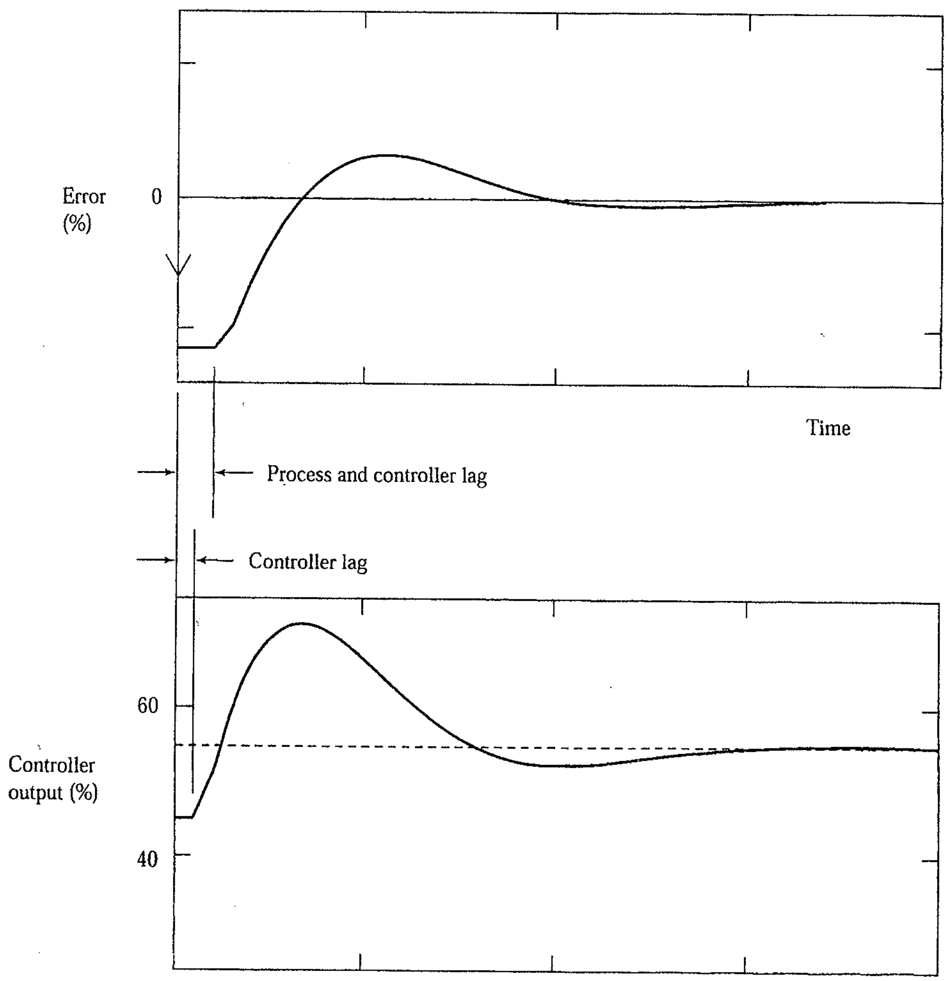

FIGURE 9.15
Illustration of integral mode output and error, showing the effect of process and control lag.

The proper valve position under the new load to maintain the constant flow rate is shown as a dashed line in the controller output graph of Figure 9.15. In the integral mode, the value initially begins to change rapidly, as predicted by Equation (9.17). As the valve opens, the error decreases and slows the valve opening rate as shown. The ultimate effect is that the system drives the error to zero at a slowing controller rate. The effect of process and control system lag is shown as simple delays in the controller output change and in the error reduction when the controller action occurs. If the process lags are too large, the error can oscillate about zero or even be cyclic. Typically, the integral mode is not used alone, but can be used for systems with small process lags and correspondingly small capacities.

9.5.3 Derivative-Control Mode

Suppose you were in charge of controlling some variable, and at some time, t_0, your helper yelled out, "The error is zero. What action do you want to take?" Well, it would seem perfectly rational to answer "None" because, after all, the error was zero. But suppose you have a screen that shows the variation of error in time and that it looks like Figure 9.16.

You can clearly see that even though the error at t_0 is zero, it is changing in time and will certainly not be zero in the following time. Therefore, some action should be taken even though the error is zero! This scenario describes the nature and need for derivative action.

Derivation controller action responds to the rate at which the error is changing—that is, the derivative of the error. Appropriately, the equation for this mode is given by the expression

$$p(t) = K_D \frac{de_p}{dt} \tag{9.18}$$

where the gain, K_D, tells us by how much percent to change the controller output for every percent-per-second rate of change of error. Derivative action is not used alone because it provides no output when the error is constant.

Derivative controller action is also called *rate action* and *anticipatory control.*

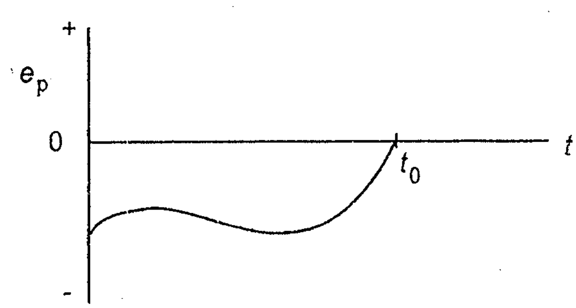

FIGURE 9.16
The error can be zero but the rate of change very large.

Figure 9.17 illustrates how derivative action changes the controller output for various rates of change of error. For this example, it is assumed that the controller output with no error or rate of change of error is 50%. When the error changes very rapidly with a positive slope, the output jumps to a large value, and when the error is not changing, the output returns to 50%. Finally, when the error is decreasing—that is, has a negative slope—the output discontinuously changes to a lower value.

The derivative mode must be used with great care and usually with a small gain, because a rapid rate of change of error can cause very large, sudden changes of controller output. Such an event can lead to instability.

Let us summarize the characteristics of the derivative mode and Equation (9.18).

1. If the error is zero, the mode provides no output.
2. If the error is constant in time, the mode provides no output.
3. If the error is changing in time, the mode contributes an output of K_D percent for every 1%-per-second rate of change of error.
4. For direct action, a positive rate of change of error produces a positive derivative mode output.

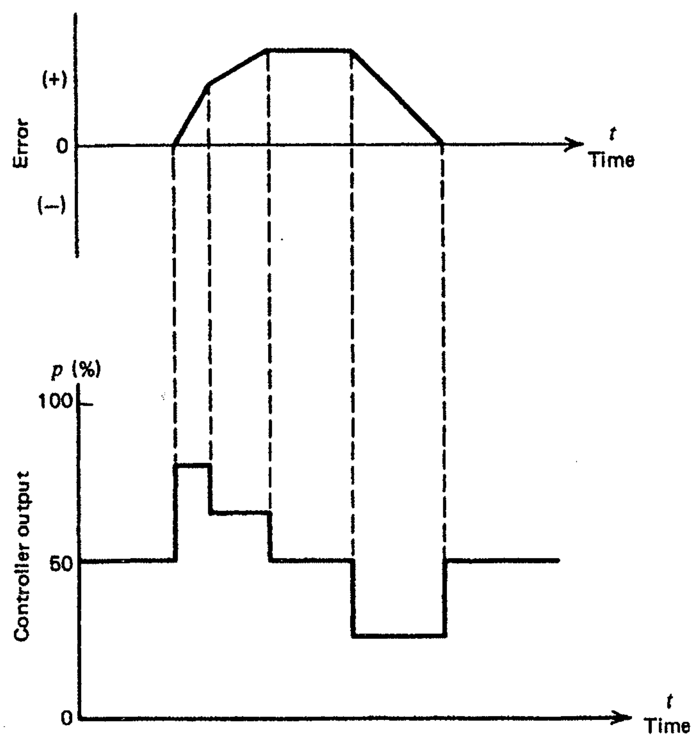

FIGURE 9.17
Derivative mode controller action changes depending on the rate of error.

9.6 COMPOSITE CONTROL MODES

It is common in the complex of industrial processes to find control requirements that do not fit the application norms of any of the previously considered controller modes. It is both possible and expedient to combine several basic modes, thereby gaining the advantages of each mode. In some cases, an added advantage is that the modes tend to eliminate some limitations they individually possess. We will consider only those combinations that are commonly used and discuss the merits of each mode.

9.6.1 Proportional-Integral Control (PI)

This is a control mode that results from a combination of the proportional mode and the integral mode. The analytic expression for this control process is found from a series combination of Equations (9.14) and (9.16):

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0) \tag{9.19}$$

where $p_I(0)$ = integral term value at $t = 0$ (initial value)

The main advantage of this composite control mode is that the one-to-one correspondence of the proportional mode is available and the integral mode eliminates the inherent offset. Notice that the proportional gain, by design, also changes the net integration mode gain, but that the integration gain, through K_I, can be independently adjusted. Recall that the proportional mode offset occurred when a load change required a new nominal controller output that could not be provided except by a fixed error from the setpoint. In the present mode, the integral function provides the required new controller output, thereby allowing the error to be zero after a load change. The integral feature effectively provides a reset of the zero error output after a load change occurs. This can be seen by the graphs of Figure 9.18. At time t_1, a load change occurs that produces the error shown. Accommodation of the new load condition requires a new controller output. We see that the controller output is provided through a sum of proportional plus integral action that finally leaves the error at zero. The proportional part is obviously just an image of the error.

Let us summarize the characteristics of the PI mode and Equation (9.19).

1. When the error is zero, the controller output is fixed at the value that the integral term had when the error went to zero. This output is given by $p_I(0)$ in Equation (9.19) simply because we chose to define the time at which observation starts as $t = 0$.
2. If the error is not zero, the proportional term contributes a correction, and the integral term begins to increase or decrease the accumulated value [initially, $p_I(0)$], depending on the sign of the error and the direct or reverse action.

The integral term cannot become negative. Thus, it will saturate at zero if the error and action try to drive the area to a net negative value.

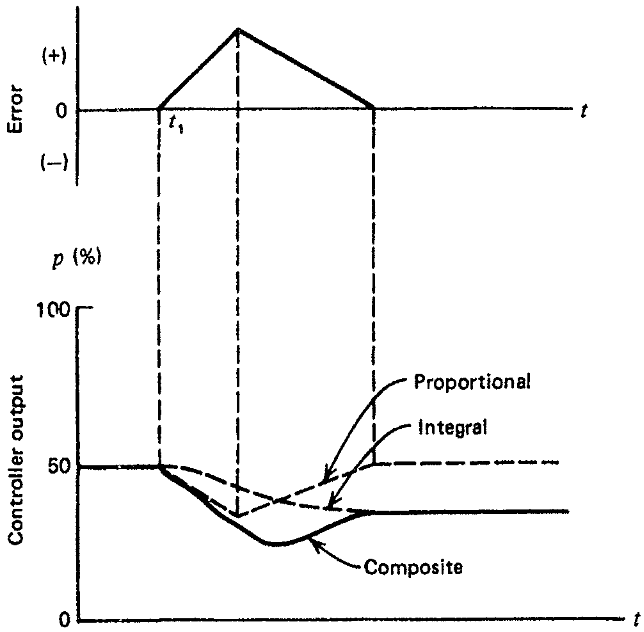

FIGURE 9.18
Proportional-integral (PI) action showing the reset action of the integral contribution. This example is for reverse action.

Application As noted, this composite proportional-integral mode eliminates the offset problem of proportional controllers. It follows that the mode can be used in systems with frequent or large load changes. Because of the integration time, however, the process must have relatively slow changes in load to prevent oscillations induced by the integral overshoot. Another disadvantage of this system is that during start-up of a batch process, the integral action causes a considerable overshoot of the error and output before settling to the operation point. This is shown in Figure 9.19, where we see the proportional band as a dashed band. The effect of the integral action can be viewed as a shifting of the whole proportional band. The proportional band is defined as that positive and negative error for which the output will be driven to 0% and 100%. Therefore, the presence of an integral accumulation changes the amount of error that will bring about such saturation by the proportional term. In Figure 9.19, the output saturates whenever the error exceeds the *PB* limits. The *PB* is constant, but its location is shifted as the integral term changes.

EXAMPLE 9.8 Given the error of Figure 9.20 (top), plot a graph of a proportional-integral controller output as a function of time.

$$K_P = 5, K_I = 1.0\ \text{s}^{-1}, \text{ and } p_I(0) = 20\%$$

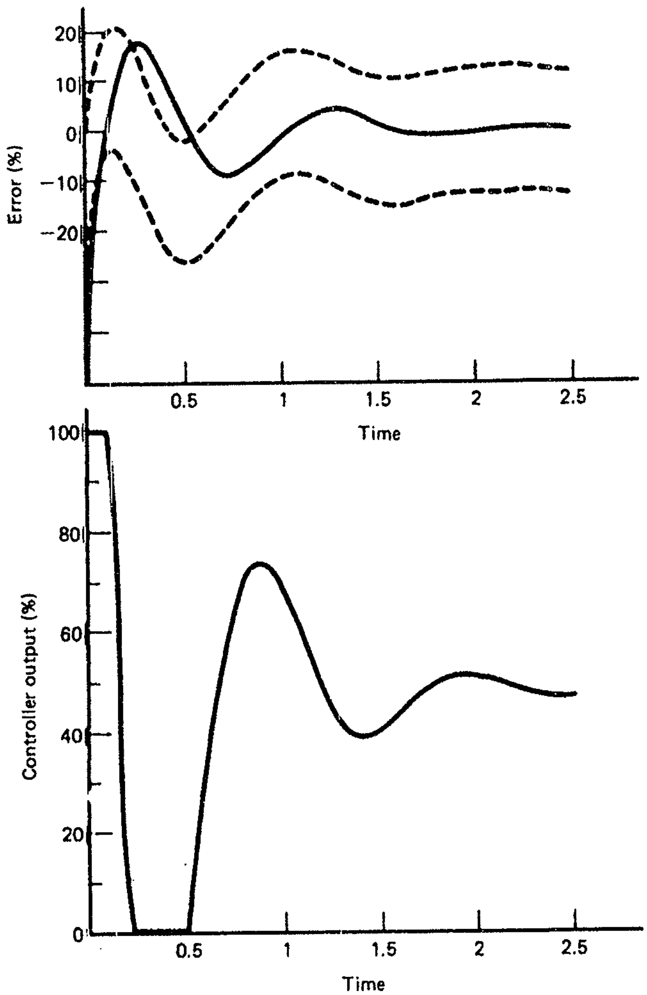

FIGURE 9.19
Overshoot and cycling often result when PI mode control is used in start-up of batch processes. The dashed lines show the proportional band.

Solution

We find the solution by an application of

$$p = K_P e_p + K_P K_I \int_0^t e_p\,dt + p_I(0)$$

To find the controller output, we solve Equation (9.19) in time. The error can be expressed in three time regions.

$$0 \leq t \leq 1 \qquad (t \text{ between 0 and 1 s})$$

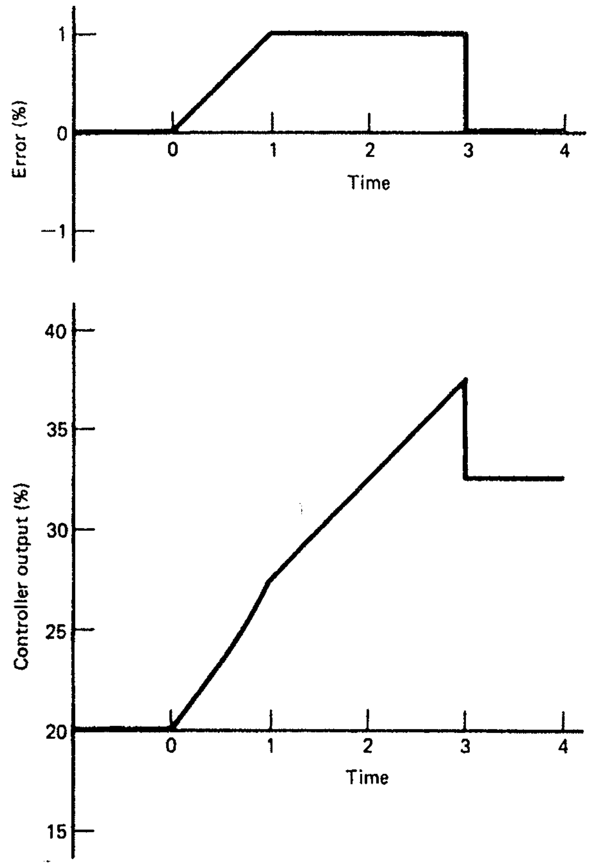

FIGURE 9.20
Solution for Example 9.8.

The error rises from 0% to 1% in 1 s. Thus, it is given by $e_p = t$.

$$1 \leq t \leq 3$$

For this time span, the error is constant and equal to 1%; therefore, it is given by $e_p = 1$.

$$t \geq 3$$

For this time, the error is zero, $e_p = 0$.

We now write out and solve Equation (9.19) for each of these time spans.

$$0 \leq t \leq 1 \qquad e_p = t$$

$$p_1 = 5t + 5\int_0^t t\,dt + 20$$

$$p_1 = 5t + 5\left[\frac{t^2}{2}\right]\int_0^t + 20$$

$$p_1 = 5t + 2.5t^2 + 20$$

This is plotted in Figure 9.20 (bottom) from 0 to 1 s. Notice the curvature because of the squared term. Remember that only the integral term accumulates values, so in finding the output at 1 s, the contribution of the proportional term, $5t$, is not included. Therefore, the starting value for the next time span is given by $p_1(1) = 2.5t^2 + 20 = 22.5\%$.

$$1 \le t \le 3 \qquad e_p = 1$$

$$p_2 = 5 + 5\int_1^t 1\,dt + 22.5$$

The integral term accumulation from 0 to 1 s forms the initial condition for this new equation.

$$p_2 = 5 + 5[t]_1^t + 22.5$$
$$p_2 = 5 + 5(t - 1) + 22.5$$

This function is plotted in Figure 9.20 from 1 to 3 s. At the end of this period, the integral term has accumulated a value of $p_2(3) = 32.5\%$.

$$t \ge 3 \qquad e_p = 0$$

$$p_3 = 5[0] + 5\int_3^t 0\,dt + 32.5$$

$$p_3 = 32.5$$

Figure 9.20 (bottom) shows that the output will stay constant at 32.5% from 3 s. The sudden drop of 5% is due to the sudden change of error from 1% to 0% at $t = 3$ s.

9.6.2 Proportional-Derivative Control Mode (PD)

A second combination of control modes has many industrial applications. It involves the serial (cascaded) use of the proportional and derivative modes. The analytic expression for this mode is found from a combination of Equations (9.14) and (9.18):

$$p = K_P e_p + K_P K_D \frac{de_p}{dt} + p_0 \tag{9.20}$$

where the terms are all defined in terms given by previous equations.

It is clear that this system cannot eliminate the offset of proportional controllers. It can, however, handle fast process load changes as long as the load change offset error is acceptable. An example of the operation of this mode for a hypothetical load change is shown in Figure 9.21. Note the effect of derivative action in moving the controller output in relation to the error rate change.

EXAMPLE 9.9 Suppose the error, Figure 9.22a, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $p_0 = 20\%$. Draw a graph of the resulting controller output.

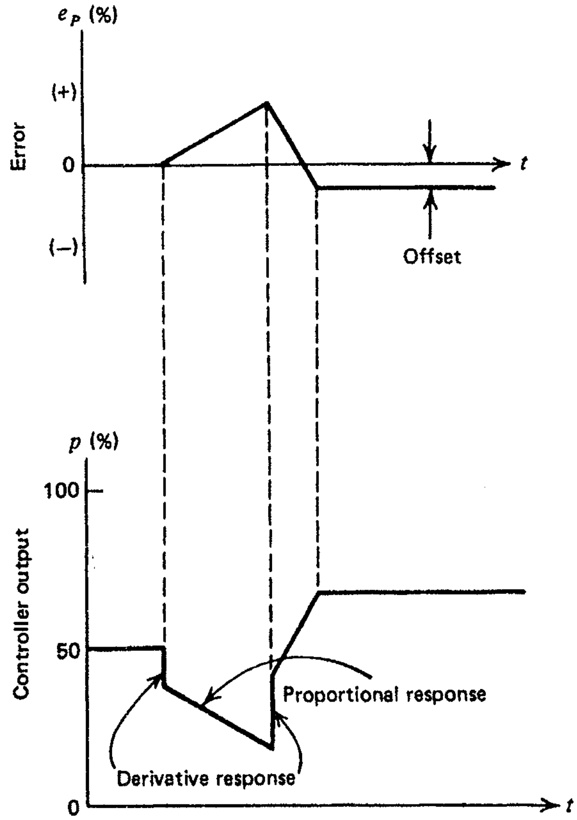

FIGURE 9.21
Proportional-derivative (PD) action showing the offset error from the proportional mode. This example is for reverse action.

Solution

In this case, we evaluate

$$p = K_P e_p + K_D K_P \frac{de_P}{dt} + p_0$$

over the two spans of the error. In the time of 0 to 1 s where $e_p = at$, we have

$$p_1 = K_P at + K_D K_P a + p_0$$

or, because $a = 1\%/\text{s}$,

$$p_1 = 5t + 2.5 + 20$$

Note the instantaneous change of 2.5% produced by this error. In the span from 1 to 3 s, we have

$$p_2 = 5 + 20 = 25$$

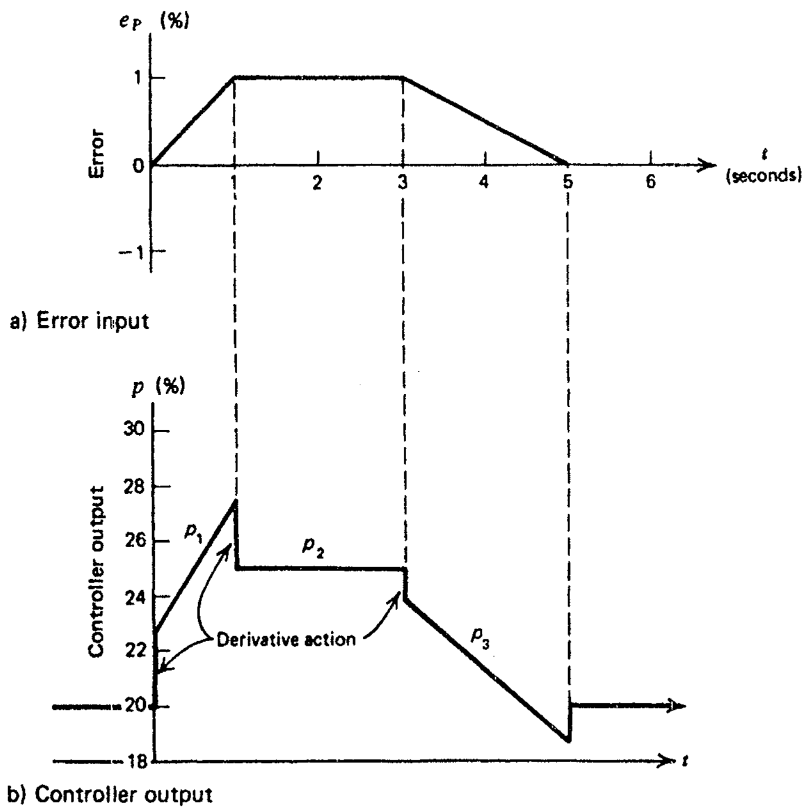

FIGURE 9.22
Solution for Example 9.9.

The span from 3 to 5 s has an error of $e_p = -0.5t + 2.5$, so that we get for 3 to 5 s

$$p_3 = -2.5t + 12.5 - 12.5 + 20$$

or

$$p_3 = -2.5t + 31.25$$

This controlled output is plotted in Figure 9.22b.

9.6.3 Three-Mode Controller (PID)

One of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes. This system can be used for virtually any process condition. The analytic expression is

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + K_P K_D \frac{de_p}{dt} + p_I(0) \qquad \textbf{(9.21)}$$

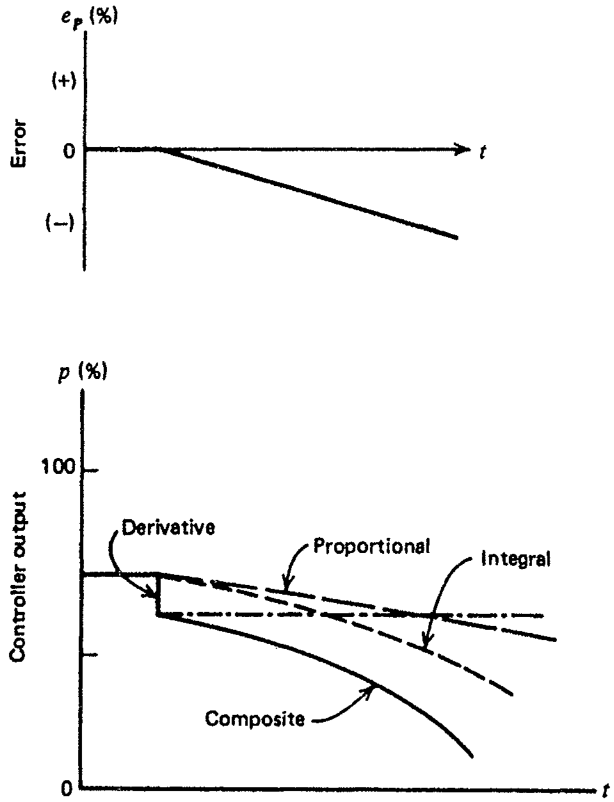

FIGURE 9.23
The three-mode controller action exhibits proportional, integral, and derivative action.

where all terms have been defined earlier.

This mode eliminates the offset of the proportional mode and still provides fast response. In Figure 9.23, the response of the three-mode system to an error is shown.

EXAMPLE 9.10

Let us combine everything and see how the error of Figure 9.22a produces an output in the three-mode controller with $K_P = 5$, $K_I = 0.7\ \text{s}^{-1}$, $K_D = 0.5$ s, and $p_I(0) = 20\%$. Draw a plot of the controller output.

Solution

From Figure 9.22a, the error can be expressed as follows:

$$
\begin{array}{ll}
0\text{–}1\text{ s} & e_p = t\% \\
1\text{–}3\text{ s} & e_p = 1\% \\
3\text{–}5\text{ s} & e_p = -\dfrac{1}{2}t + 2.5\%
\end{array}
$$

We must apply each of these spans to the three-mode equation for controller output:

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

or

$$p = 5e_p + 3.5 \int_0^t e_p \, dt + 2.5 \frac{de_p}{dt} + 20$$

From 0 to 1 s, we have

$$p_1 = 5t + 3.5 \int_0^t t \, dt + 2.5 + 20$$

or

$$p_1 = 5t + 1.75t^2 + 22.5$$

This is plotted in Figure 9.24 in the span of 0 to 1 s. At the end of 1 s, the integral term has accumulated to $p_I(1) = 21.75\%$. Now, from 1 to 3 s, we have

$$p_2 = 5 + 3.5 \int_1^t (1) \, dt + 21.75$$

or

$$p_2 = 3.5(t - 1) + 26.75$$

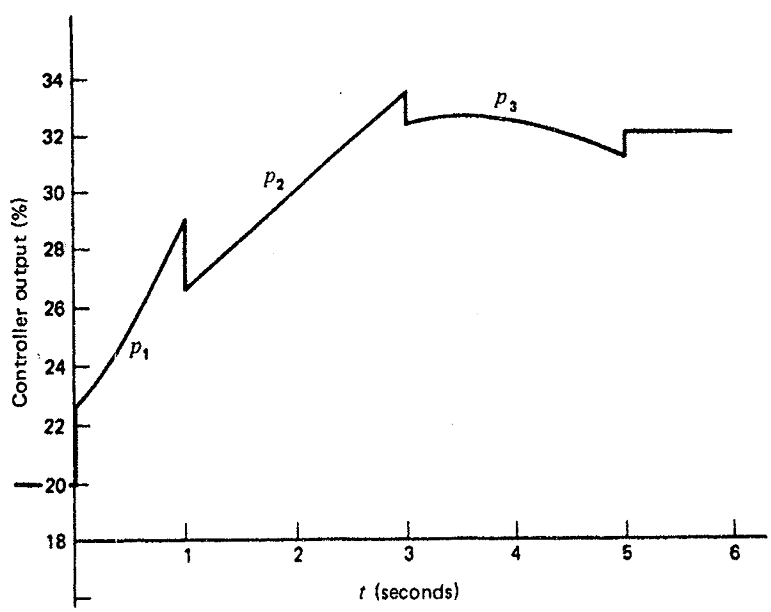

FIGURE 9.24
Solution for Example 9.10.

This controller variation is shown in Figure 9.24 from 1 to 3 s. At the end of 3 s, the integral term has accumulated to a value of $p_I(3) = 28.75\%$. Finally, from 3 to 5 s, we have

$$p_3 = 5\left(-\frac{1}{2}t + 2.5\right) + 3.5\int_3^t \left(-\frac{1}{2}t + 2.5\right) dt - \frac{2.5}{2} + 28.75$$

or

$$p_3 = -0.875t^2 + 6.25t + 21.625$$

This is plotted in Figure 9.24 from 3 to 5 s. After 5 s, the error is zero. Therefore, the output will simply be the accumulated integral response providing a constant output of $p_I = 32.25\%$.

The examples used in this chapter are idealized in terms of the sudden way that errors change. In the real world, changes are not instantaneous, and therefore the sharp breaks in output, such as those shown in Figure 9.24, do not occur.

9.6.4 Special Terminology

A number of special terms are used in process control for discussing the controller modes. The following summary defines some of these terms and shows how they relate to the equations presented in this chapter.

1. *Proportional band (PB)* Although this term was defined earlier, let us note again that this is the percentage error that results in a 100% change in controller output.
2. *Repeats per minute* This term is another expression of the integral gain for PI and PID controller modes. The term derives from the observation that the integral gain, K_I, has the effect of causing the controller output to change every unit time by the proportional mode amount. You can also see this by taking the derivative of the integral term in the controller equation. This gives a change in controller output Δp of

$$\Delta p = K_I K_P e_p \Delta t$$

   Because $K_P e_p$ is just the proportional contribution, in a unit time interval $\Delta t = 1$, K_I just repeats the proportional term. For example, if $e_p = 0.5\%$ and $K_P = 10\%$, then $K_P e_p = 5\%$. If $K_I = 10\%/(\%\text{-min})$, then every minute, the output would increase by 5% times 10%/(%-min) or 50% or "10 repeats per minute." It repeats the proportional amount 10 times per minute.
3. *Rate gain* This is just another way of saying the derivative gain, K_D. Because K_D has the units of %-s/% (or %-min/%), one often expresses the gain as time directly. Thus, a rate gain of 0.05 min or a *derivative time* of 0.05 min both mean $K_D = 0.05\%\text{-min}/\%$.
4. *Direct/reverse action* This specifies whether the controller output should increase (direct) or decrease (reverse) for an increasing controlled variable. The action is specified by the sign of the proportional gain; $K_P < 0$ is direct, and $K_P > 0$ is reverse.

SUMMARY

This chapter covers the general characteristics of controller operating modes without considering implementation of these functions. Numerous terms that are important to an understanding of controller operations are defined. The highlighted items are as follows:

1. In considering controller operating modes, it is important to know the *process load,* which is the nominal value of all process parameters, and the *process lag,* which represents a delay in reaction of the controller variable to a change of load variable.
2. Some processes exhibit *self-regulation*—that is, the characteristic that a dynamic variable adopts some nominal value commensurate with the load with no control action.
3. The controller operation is defined through a relationship between percentage *error* or *deviation* relative to full scale

$$e_p = \frac{r - b}{b_{\max} - b_{\min}} \times 100 \qquad (9.3)$$

   and the controller output as a percentage of the controlling parameter

$$p = \frac{u - u_{\min}}{u_{\max} - u_{\min}} \times 100 \qquad (9.4)$$

4. Control lag and dead time, respectively, refer to a delay in controller response when a deviation occurs and a period of no response of the process to a change in the controlling variable.
5. Discontinuous controller modes refer to instances where the controller output does not change smoothly for input error. Examples are two-position, multiposition, and floating.
6. Continuous controller modes are modes where the controller output is a smooth function of the error input or rate of change. Examples are proportional, integral, and derivative modes.
7. Composite controller modes combine the continuous modes. Examples are the proportional-integral (PI), the proportional-derivative (PD), and the proportional-integral-derivative (PID) (or three-mode).