

Graphs of Functions and Their Derivatives

Your report should consist of carefully constructed graphs and complete and clear answers to the given questions. This problem is adapted from one published in the *Resources for Calculus* series.

1. Find a polynomial function $p(x)$ of degree at least four whose graph crosses the x -axis at least four times for $x \in [-3, 3]$. *Hint: You can create a polynomial with zeros at, say, $x = -2$, $x = 0$ and $x = 1$ by writing it in the factored form $p(x) = (x + 2)x(x - 1)$. If the y -values vary too much, try multiplying the polynomial by a constant between zero and one in order to compress the y -values.*
2. Find the derivative $p'(x)$.
3. Use a graphing calculator to draw the graphs of both $p(x)$ and $p'(x)$ on the same coordinate system. Be sure to scale your graph so that all of the interesting regions of both functions are fully visible.
4. Use your graphs to answer the following questions:
 - (a) List all x -values where $p'(x) = 0$.
 - (b) List all x -values where the graph of $y = p(x)$ is at the top of a peak or at the bottom of a valley.
 - (c) Make a conjecture about graphs, functions, and derivatives based on the last two questions.
 - (d) List all intervals over which the graph of $y = p(x)$ seems to be rising as one moves from left to right.
 - (e) List all intervals over which the graph of $y = p'(x)$ is above the x -axis.
 - (f) Make a conjecture about graphs, functions, and derivatives based on the last two questions.
 - (g) List all intervals over which the graph of $y = p(x)$ seems to be falling as one moves from left to right.
 - (h) List all intervals over which the graph of $y = p'(x)$ is below the x -axis.
 - (i) Make a conjecture about graphs, functions, and derivatives based on the last two questions.
 - (j) Write a summary of what you have learned in this exercise. Be sure to include your graphs as illustrations.