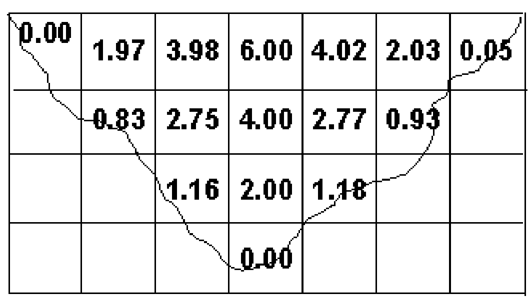
The River Trip

Dr. Zee and his talented assistant were enjoying a whitewater-rafting trip down a scenic river when they came upon a section of the river that was moving slowly. “I wonder what the flow of this river is,” mused Dr. Zee.

“Well,” answered his assistant, “I just happened to have this hydroflowmeter, and we could take velocity readings of the water flow here.”

“Let’s do that,” said Dr. Zee as he picked up a notepad and pencil to record the measurements. The two scientists created the following table of velocity readings (ft/sec) where each square on the grip represents an 8 fett by 8 fett square in a cross section of the river.



“The second square from the left in the top of the diagram represents an area of $8 \cdot 8 = 64$ square feet,” explained Dr. Zees assistant, “The velocity is 1.97 feet per second, so $8 \cdot 8 \cdots 1.97 = 126.08$ cubic feet must pass over this square during a time interval of one second.”

“Yes,” responded Dr. Zee, “adding terms like that for each of our squares we get

$$\begin{aligned}\sum_{i=1}^{16} \nu_i \Delta A &= 64 \sum_{i=1}^{16} \nu_i \\ &= 64(0.00 + 1.97 + 3.98 + \cdots + 0.00) \\ &= 21500,\end{aligned}$$

where ΔA represents the area of each square, namely 64, and where ν_i represents the water velocity in square i."

"That sum reminds me of a Riemann sum," ocserved Dr. Zee's assistant.

"Yes," agreed Dr. Zee, "if we let R represent our cross section of the river, and if for each $(x, y) \in R$ we let $\nu(x, y)$ represent water velocity at (x, y), our grid represents a partition P of R. Picking point (x, y) in each of the n rectangles in the partition and letting the norm of this partition go to zero we get the double integral

$$\iint_R \nu(x, y) dx\, dy = \lim_{\|P\| \to 0} \sum_{i=1}^{n} \nu(x, y) \Delta A."$$

"What an interesting application of integrals," exclaimed Dr. Zee.

"Yes," said his assistant, "but we are coming upon some serious rapids..."