Numerical Integration Formulas

The trapezoidal rule and Simpson’s rule provide tools for approximating
\[ \int_a^b f(x) \, dx \]
that are especially useful when it is inconvenient or impossible to find an antiderivative for \( f(x) \). Both methods involve subdividing \([a, b]\) into \( n \) subintervals of equal length with the following partition:
\[ a = x_0 < x_1 < \cdots < x_n = b. \]

The Trapezoidal Rule

Formula for Hand Calculations:
\[ \int_a^b f(x) \, dx \approx \frac{b - a}{2n} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \]

Formula for Graphics Calculators:
\[ \int_a^b f(x) \, dx \approx \frac{b - a}{2n} \left[ f(a) + f(b) + 2 \sum_{i=1}^{n-1} f \left( a + \frac{ib - a}{n} \right) \right] \]

Error Formula:
\[ |\text{Error}| \leq \frac{(b - a)^3}{12n^2} \max |f''(x)|, \quad a \leq x \leq b \]

Simpson’s Rule

Note that \( n \) must be even for Simpson’s rule. Formula for Hand Calculations:
\[ \int_a^b f(x) \, dx \approx \frac{b - a}{3n} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right] \]

Formula for Graphics Calculators:
\[ \int_a^b f(x) \, dx \approx \frac{b - a}{3n} \left[ f(a) + f(b) + 2 \sum_{i=1}^{n-1} f \left( a + (2i) \frac{b - a}{n} \right) + 4 \sum_{i=1}^{\frac{n}{2}} f \left( a + (2i - 1) \frac{b - a}{n} \right) \right] \]

Error Formula:
\[ |\text{Error}| \leq \frac{(b - a)^5}{180n^4} \max |f^{(4)}(x)|, \quad a \leq x \leq b \]