Numerical Integration Formulas

The trapezoidal rule and Simpson's rule provide tools for approximating

$$\int_a^b f(x)\,dx$$

that are especially useful when it is inconvenient or impossible to find an antiderivative for $f(x)$. Both methods involve subdividing $[a, b]$ into n subintervals of equal length with the following partition:

$$a = x_0 < x_1 < \cdots < x_n = b.$$

The Trapezoidal Rule

Formula for Hand Calculations:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$$

Formula for Graphics Calculators:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(a) + f(b) + 2\sum_{i=1}^{n-1} f\left(a + i\frac{b-a}{n}\right)\right)$$

Error Formula:

$$|\text{Error}| \le \frac{(b-a)^3}{12n^2}\max|f''(x)|,\ a \le x \le b$$

Simpson's Rule

Note that n must be even for Simpson's rule. Formula for Hand Calculations:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

Formula for Graphics Calculators:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left(f(a) + f(b) + 2\sum_{i=1}^{\frac{n}{2}-1} f\left(a + (2i)\frac{b-a}{n}\right) + 4\sum_{i=1}^{\frac{n}{2}} f\left(a + (2i-1)\frac{b-a}{n}\right)\right)$$

Error Formula:

$$|\text{Error}| \le \frac{(b-a)^5}{180n^4}\max|f^{(4)}(x)|,\ a \le x \le b$$