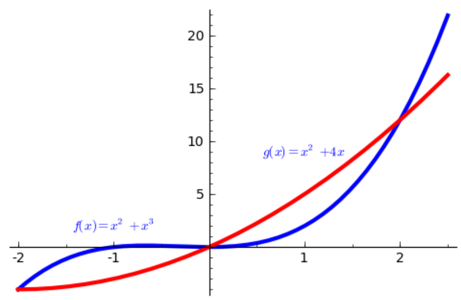
Logarithmic, Exponential, and Polynomial Functions

When the graph of $y = f(x)$ lies above the graph of $y = g(x)$ for all sufficiently large x, we say that $f(x)$ is eventually above $g(x)$. For example, in the graph below, (blue) $f(x)$ is eventually above (red) $g(x)$ since $f(x) \geq g(x)$ for all $x \geq 2$.



1. For each of the following pairs of functions, determine which function is eventually above the other. Use your calculators, and carefully draw graphs to make your case. You will need to choose an appropriate graphing window, and finding intersection points via the solve function could be helpful.

   (a) $f(x) = \ln(10x),\ g(x) = 0.5x - 10$

   (b) $f(x) = \ln(100x),\ g(x) = 0.1x$

   (c) $f(x) = \ln(1000x),\ g(x) = 0.01x$

   (d) Make a conjecture about the graphs of $f(x) = \ln(ax)$ and $g(x) = mx + b$ for $x > 0$ where a and m are positive constants.

2. Repeat the previous problem for the following pairs of functions:

   (a) $f(x) = 10x^2 + x + 10,\ g(x) = 0, 5e^x$

   (b) $f(x) = 5x^3 + x^2 + x,\ g(x) = 0, 1e^x$

   (c) $f(x) = 50x^4 + x^3 + 7x,\ g(x) = 0.1e^{0.5x}$

   (d) Make a conjecture about the graphs of $f(x)$, a polynomial function, and the graph of the exponential function $g(x) = me^{bx}$, where m and b are positive consgtants.

3. Finally, compare the following families of functions: linear functions, quadratic functions, logarithmic functions, exponential functions, and polynomial functions of degree more than 2.