10 pts  Phasor and Thévenin Analysis
Find the Thévenin voltage and Thévenin impedance between points $a$ and $b$. Express your answers in rectangular, polar, and sinusoidal forms.

$$Z_{th} = (10 + j5) \parallel (-j5)$$
$$= \frac{(10 + j5)(-j5)}{10 + j5 - j5}$$
$$= \frac{-50}{20 + j0}$$
$$= 2.5 - j5$$
$$= 5.59 \angle -63.43^\circ \ \Omega$$

$$V_{th} = I \ Z_{th}$$
$$= 3/30^\circ \times 5.59 \angle -63.43^\circ$$
$$= 16.77 \angle -33.43^\circ \ \text{V}$$
$$= 16.77 \cos (\omega t - 33.43^\circ) \ \text{V}$$
10 pts  Second Order Systems

Using the following parameters, find \( v \) for \( t \geq 0 \) seconds:

- \( C = 0.01 \text{F} \)
- \( L = 0.1 \text{H} \)
- \( R = 1.0 \text{\Omega} \)
- \( v(0) = 5.0 \text{V} \)
- \( i_d(0) = 1.0 \text{A} \)

Parallel RLC circuit

\[
\alpha = \frac{1}{2RC} = 50 \\
\omega_0 = \frac{1}{\sqrt{LC}} = 31.6228
\]

\( \alpha > \omega_0 \) so overdamped

\[
S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50 \pm 38.7298
\]

\[
= -11.27 \pm 98.73i
\]

\[
v(t) = A_1 e^{-11.27t} + A_2 e^{-98.73t}
\]

@ \( t = 0 \), \( v(0) = 5 = A_1 + A_2 \) \( \text{eq. 1} \)

KVL @ \( t = 0 \) : \( \dot{v}_C + \dot{v}_L + \dot{v}_R = 0 \)

\[
C \frac{dv_C}{dt} + 1 + \frac{5}{1} = 0
\]

\[
C \left[ A_1 (-11.27) - 98.73 A_2 \right] + 6 = 0
\]

\[
11.27 A_1 + 98.73 A_2 = 6 \text{\Omega} \text{\eq 2}
\]

Solving \( \text{eq. 1} + \text{eq. 2} \):

\[
A_1 = -2.0185
\]

\[
A_2 = 7.0185
\]

\[
v(t) = -2.0185 e^{-11.27t} + 7.0185 e^{-98.73t} \text{\quad } v \geq 0
\]
6 pts  AC Power Systems
Calculate the power factor as seen by the source.
Indicate whether the PF is leading or lagging.
Note – the source is sinusoidal.

\[
PF = \cos(\Theta_v - \Theta_i)
\]
also, \( \Theta_z = \Theta_v - \Theta_i \)
So we can find \( Z_{eq} \) as seen by the source

\[
Z_{eq} = \frac{10 + j4}{11(8-j6)}
\]
\[
= 1.088 + 4.47j = 12.635 / 20.618^\circ \Omega
\]

\[
PF = \cos(20.618^\circ)
\]
\[
= 0.936 \text{ lagging since } Z_{eq} \text{ is positive.}
\]
First-Order Circuits
The switch has been in position a for a long time. At $t = 0$ seconds, the switch moves to position b.

3 pts  Find the capacitor voltage at $t = 0$ seconds.

\[ V_C = V_A = 9 \text{V} \times \left( 15 \text{mA} \right) = 6.0 \text{V} \]

\[ V_{C(0)} = 6.0 \text{V} \]

2 pts  Find the time constant ($\tau$) of this circuit for $t > 0$ seconds.

\[ \tau = RC = 250 \times 25 \mu \text{A} = 6.25 \text{ms} \]

1 pt  Find the voltage across the capacitor at time infinity, i.e. $V_C(\infty)$.

\[ V_{C(\infty)} = 10 \text{V} \]

2 pts  Find $V_C(t)$ for $t \geq 0$ seconds.

\[ V_C(t) = V_C(\infty) + \left[ V_C(0) - V_C(\infty) \right] e^{-t/RC} \]

\[ = 10 + (6-10)e^{-t/6.25 \text{ms}} \]

\[ V_C(t) = 10 - 4e^{-t/60} \text{V} \]
Phasor Analysis

6 pts  Find the voltage across the inductor and express it in polar, rectangular, and sinusoidal forms.

\[
\begin{align*}
\dot{V}_L &= \frac{3 \cos(100t) A}{\frac{1}{jwc} + jWL + R} \\
&= \frac{3 \cos(100t)}{-100j + 100j + 100} \\
&= 3 \cos(100t) A
\end{align*}
\]

\[
\begin{align*}
V_L &= \dot{V}_L \text{ } \text{ } \text{L} \\
&= (3 \cos(100t)) \text{ } (100 \angle 90^\circ) \\
&= 300 \cos(100t + 0^\circ) V
\end{align*}
\]

2 pts  Find the current through the capacitor and express it in polar form only.

\[
\begin{align*}
\dot{i}_C &= \dot{i} - \dot{i}_L \\
&= 3 \cos(100t) - 3 \cos(90^\circ) \\
&= 3 + 3j
\end{align*}
\]

\[
\dot{i}_C = 4.243 \angle 45^\circ A
\]