

NAME KEY

SCORE _____ /60

- This is a closed book test with the exception of your calculator and three cheat sheets.
- There are six main problems, each worth ten points with the points per problem being allocated as indicated.
- Grading considerations include:
 - Your problem solution approach;
 - The difficulty of following your solution;
 - Identifying your final answer properly;
 - Placing units on your final answers.

10 pts Phasor and Thévenin Analysis

Find the Thévenin voltage and Thévenin impedance between points *a* and *b*. Express your answers in *rectangular*, *polar*, and *sinusoidal* forms.

$$Z_{th} = (10 + j5) \parallel -j5$$

$$= \frac{(10 + j5)(-j5)}{10 + j5 + j5}$$

$$Z_{th} = 2.5 - 5j \quad \Omega$$

$$= 5.59 \angle -63.43^\circ \quad \Omega$$

$$= 5.59 \cos(\omega t - 63.43^\circ) \quad \Omega$$

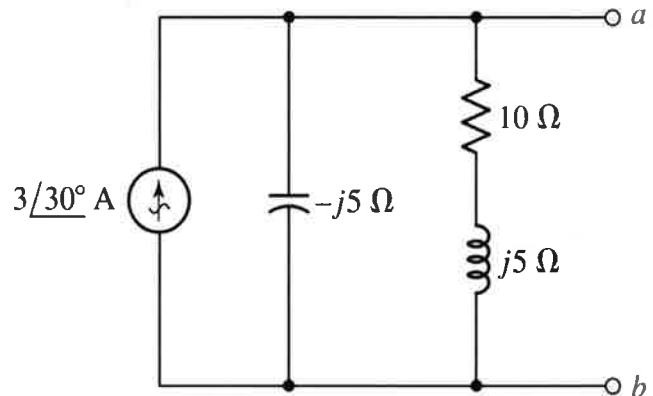
$$V_{th} = I Z_{th}$$

$$= 3 \angle 30^\circ (5.59 \angle -63.43^\circ)$$

$$V_{Th} = 16.77 \angle -33.43^\circ \quad V$$

$$= 14.0 - 9.24j \quad V$$

$$= 16.77 \cos(\omega t - 33.43^\circ) \quad V$$



10 pts Second Order Systems

Using the following parameters, find v for $t \geq 0$ seconds:

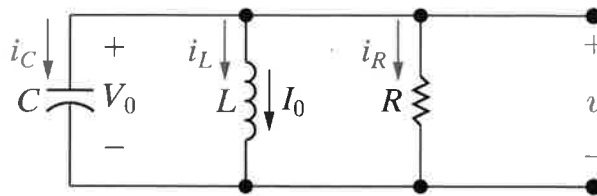
$C = 0.01\text{F}$

$L = 0.1\text{H}$

$R = 1.0\ \Omega$

$v(0) = 5.0\text{V}$

$i_L(0) = 1.0\text{A}$



Parallel RLC circuit

$$\alpha = \frac{1}{2RC} = 50$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 31.6228$$

 $\alpha > \omega_0$ so overdamped

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -50 \pm 38.7298$$

$$= -11.27, -88.73$$

$$\alpha = 50$$

$$\omega_0 = 31.6228$$

$$s_1 = -11.2702$$

$$s_2 = -88.7298$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$= A_1 e^{-11.27t} + A_2 e^{-88.73t}$$

$$\text{at } t=0, v(0) = 5 = A_1 + A_2 \quad (1)$$

$$\text{KVL at } t=0 : i_C + i_L + i_R = 0$$

$$C \frac{dv_C}{dt} + 1 + \frac{5}{1} = 0$$

$$C [A_1(-11.27) - 88.73 A_2] + 6 = 0$$

$$11.27 A_1 + 88.73 A_2 = 600 \quad (2)$$

$$\text{Solving } (1) + (2) : A_1 = -2.0185$$

$$A_2 = 7.0185$$

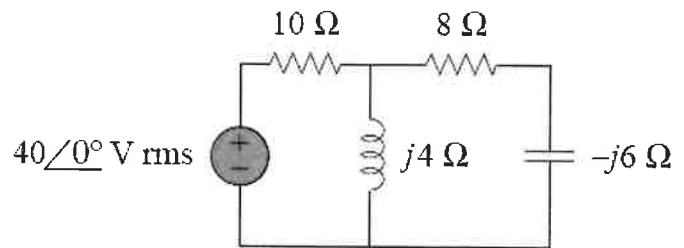
$$v(t) = -2.019 e^{-11.27t} + 7.019 e^{-88.73t} \text{ V } \quad t \geq 0$$

6 pts AC Power Systems

Calculate the power factor as seen by the source.

Indicate whether the PF is leading or lagging.

Note - the source is sinusoidal.



$$PF = \cos(\theta_v - \theta_i)$$

also, $\theta_z = \theta_v - \theta_i$ so

we can find z_{eq} as seen by the source

$$z_{eq} = 10 + j4 \parallel (8 - j6)$$

$$= 11.88 + 4.47j = 12.615 \angle 20.618^\circ \Omega$$

$$PF = \cos(20.618^\circ)$$

$$= 0.936 \text{ lagging since } \angle \text{ of } z \text{ is positive.}$$

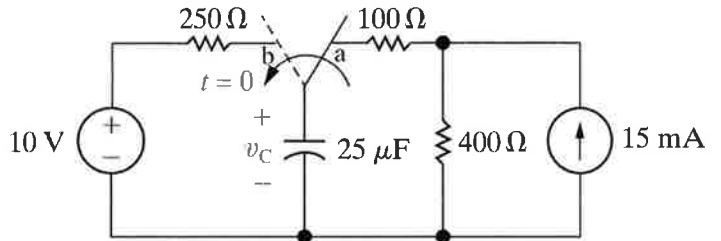
NAME KEY

SCORE _____/30

This test is 60 minutes in length and is closed book except for your calculator and one side of a cheat sheet.

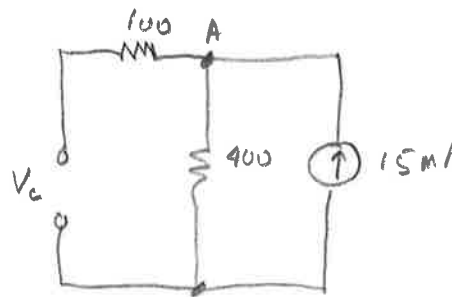
First-Order Circuits

The switch has been in position **a** for a long time. At $t = 0$ seconds, the switch moves to position **b**.



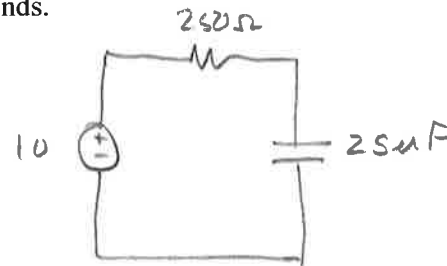
3 pts Find the capacitor voltage at $t = 0$ seconds.

$$\begin{aligned}
 V_C &= V_A \\
 &= 400(15\text{mA}) \\
 &= 6.0\text{V} \\
 \boxed{V_C(0) = 6.0\text{V}}
 \end{aligned}$$



2 pts Find the time constant (τ) of this circuit for $t > 0$ seconds.

$$\begin{aligned}
 \tau &= RC \\
 &= 250(25\mu\text{F}) \\
 \boxed{\tau = 6.25\text{ms}}
 \end{aligned}$$



1 pt Find the voltage across the capacitor at time infinity, i.e. $V_C(\infty)$.

$$\boxed{V_C(\infty) = 10\text{V}}$$

2 pts Find $V_C(t)$ for $t \geq 0$ seconds.

$$\begin{aligned}
 V_C(t) &= V_C(\infty) + [V_C(0) - V_C(\infty)] e^{-t/RC} \\
 &= 10 + (6 - 10)e^{-t/RC} \\
 \boxed{V_C(t) = 10 - 4e^{-160t} \text{ V}}
 \end{aligned}$$

Phasor Analysis

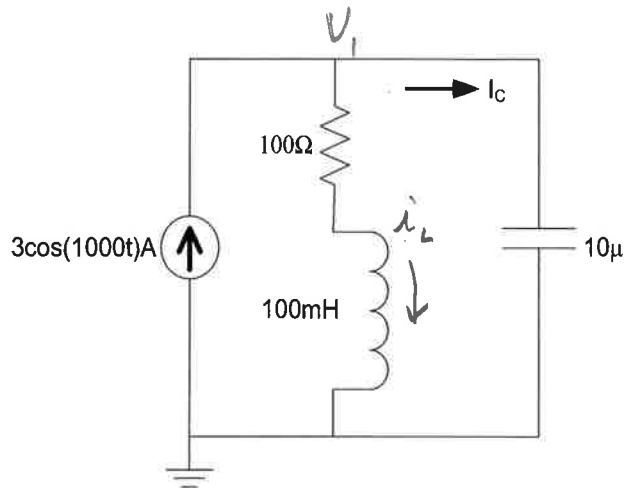
6 pts Find the voltage across the inductor and express it in **polar**, **rectangular**, and **sinusoidal** forms.

Current divider:

$$\begin{aligned} i_L &= \frac{3 \angle 0}{\frac{1}{j\omega C} + j\omega L + R} \\ &= \frac{3 \angle 0 (100 \angle -90^\circ)}{-100j + 100j + 100} \\ &= 3 \angle -90^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} V_L &= i_L Z_L \\ &= (3 \angle -90^\circ)(100 \angle 90^\circ) \end{aligned}$$

$$\begin{aligned} V_L &= 300 \angle 0^\circ \text{ V} \\ &= 300 + 0j \text{ V} \\ &= 300 \cos(1000t + 0^\circ) \text{ V} \end{aligned}$$



$$\begin{aligned} V_L &= 3 \angle -90^\circ (100 + 100j) \\ &= 424.2 \angle -45^\circ \\ &= 300 - 300j \end{aligned}$$

2 pts Find the current through the capacitor and express it in **polar** form only.

$$\begin{aligned} i_C &= i - i_L \\ &= 3 \angle 0^\circ - 3 \angle -90^\circ \\ &= 3 + 3j \end{aligned}$$

$$i_C = 4.243 \angle 45^\circ \text{ A}$$