Sections 7.1,2 Objective

• Review sinusoidal representation and complex math.
Properties of a Sinusoidal Waveform

The general form of sinusoidal wave is

\[ v(t) = V_m \sin(\omega t + \theta) \]

where:
- \( V_m \) is the amplitude in peak voltage;
- \( \omega \) is the angular frequency in radian/second, also \( 2\pi f \);
- \( \theta \) is the phase shift in degrees or radians.

Frequency Review

Period \( \approx 6.28 \) seconds, Frequency = \( 0.1592 \) Hz
Amplitude Review

Peak: Blue 1 volt, Red 0.8 volts
Peak-to-Peak: Blue 2 volts, Red 1.6 volts
Average: 0 volts

Phase Shift Review

\[ y_{\text{blue}} = \sin(t) \]
\[ y_{\text{red}} = 0.8 \sin(t + 1) \]

Red leads Blue by 57.3 degrees (1 radian) \[ \phi = \frac{1}{6.28} \times 360^\circ = 57.3^\circ \]
More on Phase

- The red wave \([V_m \sin(\omega t + \theta)]\) \textbf{leads} the wave in green by \(\theta\);
- The green wave \([V_m \sin(\omega t)]\) \textbf{lags} the wave in red by \(\theta\);
- The units of \(\theta\) and \(\omega t\) must be consistent.

Basic AC Circuit Components

- AC Voltage and Current Sources \textbf{(active components)}
- Resistors (R)
- Inductors (L) \textbf{(passive components)}
- Capacitors (C)
- Inductors and capacitors have limited energy storage capability.
**AC Voltage and Current Sources**

Voltage Sources

Current Sources

Amplitude = 10\(V_{\text{peak}}\)
\(\omega = 2\pi\) so \(F = 1\text{Hz}\)
Phase shift = 45°

**Sinusoidal Steady State (SSS) Analysis**

- SSS is important for circuits containing capacitors and inductors because these elements provide little value in circuits with only DC sources;
- Sinusoidal means that source excitations have the form \(V_S \cos(\omega t + \theta)\) or \(V_S \sin(\omega t + \theta)\);
- Since \(V_S \sin(\omega t + \theta)\) can be written as \(V_S \cos(\omega t + \theta - \pi/2)\), we will use \(V_S \cos(\omega t + \theta)\) as the general form for our source excitation;
- Steady state means that all transient behavior in the circuit has decayed to zero.
Sinusoidal Steady State Response

The SSS response of a circuit to a sinusoidal input is also a sinusoidal signal \textit{with the same frequency} but with possibly different amplitude and phase shift.

\[ v_1(t) \cos \text{ wave} \quad v_2(t) \cos \text{ wave} \quad i(t) \cos \text{ wave} \quad v_L(t) \cos \text{ wave} \]

Review of Complex Numbers

- Complex numbers can be viewed as vectors where the X-axis represents the real part and the Y-axis represents the imaginary part.
- There are two common ways to represent complex numbers:
  - Rectangular form: \( 4 + j3 \)
  - Polar form: \( 5 \angle 37^\circ \)
Complex Number Forms

Rectangular form: \( a + jb \)

Polar form: \( \rho \angle \theta \)

\[ \begin{align*}
\rho &= \sqrt{a^2 + b^2} \\
\theta &= \arctan \left( \frac{b}{a} \right) \\
a &= \rho \cos \theta \\
b &= \rho \sin \theta
\end{align*} \]

Complex Math – Rectangular Form

\[ p = a + jb \quad q = c + jd \]

- Addition and subtraction

\[ \begin{align*}
x &= p + q = (a + c) + j(b + d) \\
y &= p - q = (a - c) + j(b - d)
\end{align*} \]

- Example

\[ p = 3 + j4 \quad q = 1 - j2 \]

\[ \begin{align*}
x &= p + q = (3 + 1) + j(4 - 2) = 4 + j2 \\
y &= p - q = (3 - 1) + j(4 - (-2)) = 2 + j6
\end{align*} \]
Complex Math – Rectangular Form

\[ p = a + jb \quad q = c + jd \]

\[ \cdot \text{ Multiplication (easier in polar form)} \]

\[ x = p \times q = ac + jad + jbc + j^2bd = (ac - bd) + j(ad + bc) \]

\[ \cdot \text{ Example} \]

\[ p = 3 + j4 \quad q = 1 - j2 \]

\[ x = p \times q = [(3)(1) - (4)(-2)] + j[(3)(-2) + (4)(1)] \]

\[ = 11 - j2 \]

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Complex Math – Rectangular Form

\[ p = a + jb \quad q = c + jd \]

\[ \cdot \text{ Division (easier in polar form)} \]

\[ x = \frac{p}{q} = \frac{a + jb}{c + jd} = \left( \frac{a + jb}{c + jd} \right) \left( \frac{c - jd}{c - jd} \right) = \left( \frac{ac + bd + j(bc - ad)}{c^2 + d^2} \right) \]

\[ \cdot \text{ Example} \]

\[ p = 3 + j4 \quad q = 1 - j2 \]

\[ x = \frac{p}{q} = \frac{(3)(1) + (4)(-2) + j((4)(1) - (3)(-2))}{1^2 + (-2)^2} = \frac{-5 + j10}{5} = -1 + j2 \]
Euler’s Identity

• Euler’s identity states that \( e^{j\theta} = \cos(\theta) + j\sin(\theta) \)
• A complex number can then be written as:
  \[ r = a + jb = \rho \cos(\theta) + j\rho \sin(\theta) = \rho [\cos(\theta) + j\sin(\theta)] = \rho e^{j\theta} \]
• Using shorthand notation, we write this as:
  \[ \rho e^{j\theta} \equiv \rho \angle \theta \]

![Diagram showing complex number in polar form]

Complex Math – Polar Form

\[ x = a + jb = \rho e^{j\theta} = \rho \angle \theta \quad \rho = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}(\frac{b}{a}) \]

\[ p = m_1 e^{j(\theta_1)} \quad q = m_2 e^{j(\theta_2)} \]

• Addition and subtraction - too hard in polar so convert to rectangular coordinates.
• Multiplication
  \[ z = p \times q = m_1 m_2 e^{j(\theta_1 + \theta_2)} \]

• Example
  \[ p = 6 e^{j\left(\frac{\pi}{6}\right)} \quad q = 2 e^{j\left(\frac{\pi}{2}\right)} \quad z = p \times q = (6)(2) e^{j\left(\frac{\pi}{6} + \frac{\pi}{2}\right)} = 12 e^{j\frac{2\pi}{3}} \]
  \[ p = 6 \angle 30^\circ \quad q = 2 \angle 90^\circ \quad z = p \times q = 12 \angle 120^\circ \]
Complex Math – Polar Form

\[ x = a + jb = re^{j\theta} = \rho \angle \theta \quad \rho = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1}\left(\frac{b}{a}\right) \]

\[ p = m_1e^{j\theta_1} \quad q = m_2e^{j\theta_2} \]

- Division
  \[ z = p \div q = \frac{m_1}{m_2} e^{j(\theta_1 - \theta_2)} \]

- Example
  \[ p = 6e^{j(\frac{\pi}{6})} \quad q = 2e^{j(\frac{\pi}{7})} \]
  \[ z = p \div q = \frac{6}{2} e^{j\left(\frac{\pi}{6} - \frac{\pi}{7}\right)} = 3e^{j\left(-\frac{\pi}{3}\right)} = 3 \angle -60^\circ \]

More on Sinusoids

- Suppose you connect a function generator to any circuit containing resistors, inductors, and capacitors. If the function generator is set to produce a sinusoidal waveform, then every voltage drop and every current in the circuit will also be a sinusoid of the same frequency. Only the amplitudes and phase angles will (may) change.

- The same thing is not necessarily true for waveforms of other shapes like triangle or square waveforms.

- Fortunately, it turns out that sinusoids are not only the easiest waveforms to work with mathematically, they're also the most useful and occur quite frequently in real-world applications.
Phasors

• A phasor is a vector that represents an AC electrical quantity such as a voltage waveform or a current waveform;
• The phasor's length represents the peak value of the voltage or current;
• The phasor's angle represents the phase angle of the voltage or current;
• Phasors are used to represent the relationship between two or more waveforms with the same frequency.

Sections 7.1,2 Summary

• Reviewed sinusoid representation and complex math.