Chapter 6
Inductance and Capacitance

Engr228
Circuit Analysis
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Chapter 6 Objectives

• Understand relationships between voltage, current, power, and energy in inductors and capacitors;
• Know that current must be continuous in an inductor and voltage must be continuous in a capacitor;
• Be able to correctly combine inductors in series and parallel to form a single equivalent inductor;
• Be able to correctly combine capacitors in series and parallel to form a single equivalent capacitor.
Inductance

- Faraday’s law states that voltage is induced in an ideal conductor if a changing current passes through it;
- The induced voltage is proportional to the time rate of change (derivative) of the current;
- As a result, energy is stored in the magnetic field surrounding the wire;
- It is customary to use the symbol $L$ for inductance. The SI unit of inductance is the **Henry** (H), named after American scientist and magnetic researcher Joseph Henry.

![Figure 7.1](image1)  Time-varying current flowing through an ideal conductor.

Real Inductors

![Real Inductors](image2)
**Inductors**

![Inductor Diagram]

- The inductor is often called a coil because physically coiling a wire greatly increases its inductance, especially if it is coiled around a magnetic material.
- The governing voltage and current relationship is:

\[ v_L(t) = L \frac{di(t)}{dt} \]

**DC Characteristics of Inductors**

![DC Inductor Circuits]

The inductor acts like a "short circuit" at DC because the time rate of current change is equal to zero.

\[ v_L(t) = L \frac{di(t)}{dt} \]
Voltage and current in a resistor are in phase as shown below. The amplitudes may vary due to Ohm’s Law, but the phase is the same for the current and the voltage.

Voltage-Current Relationship in a Resistor

Current and voltage in an inductor are not in phase with each other. For sinusoidal waves, the voltage across an inductor leads the current through it by 90°. (In other words, the current lags the voltage by 90°.) In the diagram below, the tall blue waveform represents the voltage across an inductor and the shorter purple waveform represents the current through the inductor.

Voltage-Current Relationship in an Inductor
Inductor VI Characteristics

From the circuit shown at the right, find \( i(t) \).

\[
v(t) = L \frac{di(t)}{dt} \\
i(t) = \frac{1}{L} \int v(t) dt = \frac{1}{L} \int A \sin \omega t dt \\
= \frac{A}{L} \int \sin \omega t dt = \frac{A}{L} \left( \frac{-\cos \omega t}{\omega} \right) \\
= \frac{A}{\omega L} (-\cos \omega t) = \frac{A}{\omega L} (\sin \omega t - \frac{\pi}{2})
\]

Voltage-Current Relationships

\[
i(t) = \frac{A}{\omega L} (\sin \omega t - \frac{\pi}{2}) \\
\omega L \text{ is known as the Inductive Impedance}
\]
Power and Energy in an Inductor

- Instantaneous power is measured in Watts (W):
  \[ p = v_L i_L = L_i \frac{di_L}{dt} \]

- Energy is the integral of power over a time interval and is measured in Joules (J). Energy is stored in the **Magnetic** field surrounding the inductor:
  \[ w_L(t) = \int_{t_0}^t p(t) dt \]
  \[ w_L(t) = \frac{1}{2} L_i^2 \]

Inductor Combinations

\[ L_{eq} = L_1 + L_2 + \ldots + L_N \]

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N} \]
Capacitance

- Like the inductor, the capacitor is an energy storing device;
- The *Capacitance* \( C \) is a measure of the capacitor’s potential to store energy in an electric field;
- A capacitor is constructed of two conducting plates separated by an insulator.

![Figure 7.15](image)

Real Capacitors
Practical Definition of Capacitance

• **Capacitance** is the ability of a body to hold an electrical charge. A common form of an energy storage device is the parallel-plate capacitor where the capacitance is directly proportional to the surface area of the conductor plates and inversely proportional to the separation distance between the plates. If the charges on the plates are $+Q$ and $-Q$, and $V$ gives the voltage between the plates, then the capacitance is given by:

$$C = \frac{Q}{V}$$

Capacitors

- The unit of capacitance is the Farad (F);
- 1 F = 1 Amp-Second/Volt = 1 Coulomb/Volt;
- The governing voltage and current relationship is:

$$i_C(t) = C \frac{dv_C(t)}{dt}$$
More on Capacitance

• The capacitance used in electronic circuits is typically several orders of magnitude smaller than the farad. The most common units of capacitance in use today are the microfarad (µF), nano-farad (nF), and pico-farad (pF).
• The capacitance of a parallel-plate capacitor constructed of two parallel plates, both of area $S$ separated by a distance $d$, is approximately equal to the following:

$$C = \varepsilon_r \varepsilon_0 (S/d)$$

- $C$ is the capacitance;
- $\varepsilon_r$ is the relative static permittivity (sometimes called the dielectric constant) of the material between the plates (for a vacuum, $\varepsilon_r = 1$);
- $\varepsilon_0$ is the electric constant ($\varepsilon_0 \approx 8.854 \times 10^{-12} \text{ Fm}^{-1}$);
- $S$ is the area of overlap of the two plates;
- $d$ is the separation between the plates.

DC Characteristics of a Capacitor

The capacitor acts like an “open circuit” at DC because the time rate of change of voltage is zero so, no current can flow through it.

$$i_C(t) = C \frac{dv_C(t)}{dt}$$
The impedance of the capacitor is called **Capacitive Reactance**.

**Voltage-Current Relationship in a Capacitor**

Current and voltage in a capacitor are not in phase with each other. For sinusoidal waves, the voltage across a capacitor lags the current through it by 90°. (In other words, the current leads the voltage by 90°.) In the diagram below, the tall purple waveform represents the current through a capacitor and the shorter blue waveform represents the voltage across a capacitor.
Power and Energy in a Capacitor

Power in a capacitor:

\[ p_c = v_c i_c = C v_c \frac{dv_c}{dt} \]

Energy is stored in the *Electric* field in a capacitor:

\[ w_c(t) = \int_{t_0}^t p\,dt \]

\[ w_c(t) = \frac{1}{2} C v_c^2 \]

Capacitor Combinations

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N} \]

\[ C_{eq} = C_1 + C_2 + \ldots + C_N \]
Chapter 6 Summary

- Showed how to calculate voltage, current, power, and energy in inductors and capacitors;
- Showed that current must be continuous in an inductor and voltage must be continuous in a capacitor;
- Showed how to combine inductors in series and parallel to form a single equivalent inductor;
- Showed how to combine capacitors in series and parallel to form a single equivalent capacitor.