Section 3.2 Objective

- Learn to apply the node-voltage method to analyze an electric circuit of any configuration, so long as it is linear and planar.
Resistors in Series

\[ v_x = v_1 + v_2 + \cdots + v_N \]
\[ v_x = R_1 i + R_2 i + \cdots + R_N i \]
\[ = (R_1 + R_2 + \cdots + R_N) i \]

\[ v_x = R_{eq} i \]
\[ R_{eq} = R_1 + R_2 + \cdots + R_N \]

Voltage Division

Resistors in series “share” the voltage applied to them.

\[ i = \frac{v}{R_1 + R_2} \]
\[ v_2 = i R_2 = \left( \frac{v}{R_1 + R_2} \right) R_2 \]
\[ v_2 = \frac{R_2}{R_1 + R_2} v \]
Parallel Connections

- Elements in a circuit connected head-to-head and tail-to-tail have a common voltage across them and are said to be connected in *parallel*.

\[
\begin{align*}
5 \text{ A} & \quad 10 \Omega \quad + \varepsilon \quad - \\
1 \text{ A} & \quad 10 \Omega
\end{align*}
\]

Resistors in Parallel

\[
\begin{align*}
\text{Equation:} & \quad i_s = i_1 + i_2 + \ldots + i_N \\
& \quad i_s = \frac{v}{R_1} + \frac{v}{R_2} + \ldots + \frac{v}{R_N} = \frac{v}{R_{eq}} \\
\text{Resistance:} & \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}
\end{align*}
\]
Two Resistors in Parallel

\[ R_{eq} = R_1 \parallel R_2 \]
\[ = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \]

Connecting resistors in parallel makes the equivalent resistance \textit{smaller. Always.}

Current Division

Resistors in parallel “share” the current through them.

\[ i_2 = \frac{v}{R_2} \]
\[ = \frac{i(R_1 \parallel R_2)}{R_2} \]
\[ = \frac{i}{R_2} \cdot \frac{R_1 R_2}{R_1 + R_2} \]
\[ i_2 = i \cdot \frac{R_1}{R_1 + R_2} \]
Resistance is futile
Circuit Analysis

- We need an organized method of applying KVL, KCL, and Ohm’s law;
- *Nodal* analysis assigns voltages to each node, and then we apply *Kirchhoff’s Current Law* to solve for the node voltages;
- *Mesh* analysis assigns currents to each mesh, and then we apply *Kirchhoff’s Voltage Law* to solve for the mesh currents.

Review - Nodes, Paths, Loops, Branches

- These two circuits are equivalent.
- There are three nodes and five branches:
  - *Node*: a point at which two or more elements have a common connection;
  - *Path*: a sequence of nodes;
  - *Branch*: a single path in a circuit composed of one simple element and the node at each end of that element;
  - *Loop*: a closed path.
**Node Example**

- Node = every point along the same wire

![Node Example Diagram]

3 nodes

**Nodes**

- How many nodes in the circuits below?

![Nodes Diagrams]

Reference node

Reference node

Engr228 Zybooks Section 3.2 – Nodal Analysis
Notes on Writing Nodal Equations

• All terms in the equations are in units of current;
• Everyone has their own style of writing nodal equations
  – The important thing is that you remain consistent.
• Probably the easiest method if you are just getting started is to
  remember that:

  \[ \text{current entering a node} = \text{current leaving the node} \]

• Current directions can be assigned arbitrarily, unless they are
  previously specified.

The Nodal Analysis Method

• Assign voltages to every node relative to a reference node.
Choosing the Reference Node

- By convention, the bottom node is often the reference node;
- If a ground connection is shown, then that becomes the reference node;
- Otherwise, choose a node with many connections;
- Assign the reference node a value of 0.00 volts.

Apply KCL to Find Voltages

- Assume reference voltage = 0.00 volts;
- Assign current names and directions;
- Apply KCL to node \( v_1 \) (\( \Sigma \text{out} = \Sigma \text{in} \));
- Apply Ohm’s law to each resistor.

\[
\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1
\]
Apply KCL to Find Voltages

- Apply KCL to node $v_2$ ($\Sigma$ out = $\Sigma$ in);
- Apply Ohm’s law to each resistor.

\[
\frac{v_1 - v_2}{5} = \frac{v_2 - 0}{1} + (-1.4)
\]

We now have two equations for the two unknowns $v_1$ and $v_2$ and we can solve them simultaneously:

$v_1 = 5V$ and $v_2 = 2V$

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Example Problem Page 96 (Nilsson 11th)

Solve for the node voltages in the circuit below.

Answer: Left node = 9.09 V  
Right node = 10.91 V
Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source. Key step: eliminate $i_1$ from the equations using $v_1 = 2i_1$

\[ 15 = \frac{v_1 - v_2}{1} + \frac{v_1 - 0}{2} \]

\[ \frac{v_1 - v_2}{1} + 3i_1 = \frac{v_2 - 0}{3} \]

\[ i_1 = \frac{v_1 - 0}{2} \]

*Answer: 4.5 kW being generated*

Node Voltage Example

- How many nodes are in this circuit?
- How many nodal equations must you write to solve for the unknown voltages?
**Node Voltage Example – Node V1**

\[
\begin{align*}
\text{At node } V_1 & \quad 8 + \frac{V_1 - V_3}{4} + 3 + \frac{V_1 - V_2}{3} = 0 \\
& \quad 96 + 3V_1 - 3V_3 + 36 + 4V_1 - 4V_2 = 0 \\
& \quad 7V_1 - 4V_2 - 3V_3 = -132
\end{align*}
\]

**Node Voltage Example – Node V2**

\[
\begin{align*}
\text{At node } V_2 & \quad \frac{V_2 - V_1}{3} - 3 + \frac{V_2 - V_3}{2} + \frac{V_2 - 0}{1} = 0 \\
& \quad 2V_2 - 2V_1 - 18 + 3V_2 - 3V_3 + 6V_2 = 0 \\
& \quad -2V_1 + 11V_2 - 3V_3 = 18
\end{align*}
\]
Node Voltage Example – Node V3

At node $V_3$

\[
\frac{V_3 - V_2}{2} + \frac{V_3 - V_1}{4} - 25 + \frac{V_3 - 0}{5} = 0
\]
\[
10V_3 - 10V_2 + 5V_3 - 5V_1 - 500 + 4V_3 = 0
\]
\[
-5V_1 - 10V_2 + 19V_3 = 500
\]

Node Voltage Example – Solution

\[
7V_1 - 4V_2 - 3V_3 = -132
\]
\[
-2V_1 + 11V_2 - 3V_3 = 18
\]
\[
-5V_1 - 10V_2 + 19V_3 = 500
\]

Answer:

\[
V_1 = 0.956V
\]
\[
V_2 = 10.576V
\]
\[
V_3 = 32.132V
\]
Section 3.2 Summary

• You learned to apply the node-voltage method to analyze an electric circuit of any configuration, so long as it is linear and planar.