Chapter 2
Circuit Elements

Engr228
Circuit Analysis
Dr Curtis Nelson

Chapter 2 Objectives

• Understand symbols and behavior of the following circuit elements:
  – Independent voltage and current sources;
  – Dependent voltage and current sources;
  – Resistors.
• Be able to use Ohm’s and Kirchhoff’s laws to analyze circuits.
• Know how to calculate the power for each element in a circuit and determine whether or not the power balances.
Various Circuit Elements

- Electric sources
  - Independent Sources – voltage, current;
  - Dependent Sources – voltage, current.
- Resistors, inductors, capacitors
- Measurement devices
  - Ammeters (current);
  - Voltmeters (volts);
  - Ohmmeters (resistance).
- Electric wire

Independent Voltage Sources

- An ideal voltage source is a circuit element that will maintain the specified voltage $v_s$ across its terminals.
A Battery as an Independent Voltage Source

- An “ideal” battery is an example of an independent voltage source.
  - A “real-world” battery has a maximum power that it can deliver.

Independent Current Sources

- An ideal current source is a circuit element that maintains the specified current flow $i_s$ through its terminals.
Dependent Voltage and Current Sources

- A dependent voltage or current source establishes a voltage or current whose value depends on a voltage or current elsewhere in the circuit.

\[
v_s = \mu v_x
\]
\[
i_s = \alpha v_x
\]

(a) \hspace{2cm} (c)

\[
v_s = \rho i_x
\]
\[
i_s = \beta i_x
\]

(b) \hspace{2cm} (d)

Current

- Current is the same in all elements connected in Series.

\[
1V \quad 1\Omega
\]

\[
1A \quad 1A \quad 1A
\]
Voltage

- Voltage is the same for all elements connected in Parallel.

![Voltage Diagram]

Interconnection of Ideal Sources

- Which of the following are valid connections?

  a) Valid  
  b) Valid  
  c) Not Valid  
  d) Not Valid  
  e) Valid
**The Concept of Ground**

- *Ground* is commonly referred to as a reference point.
- *Ground* is said to be at a potential of 0.00 volts. In other words, *Ground* has zero voltage because it is referenced to itself.
- Ground symbols:

![Ground symbols](image)

**Ground**

![Ground diagram](image)
Open Circuit

- An open circuit means no current can flow \((i = 0)\).
- Voltage across an open circuit can be any value.
- An open circuit is equivalent to a resistance of \(\infty \, \Omega\).
- Open circuit summary:
  - Infinite resistance;
  - Zero current;
  - Voltage can be any value.

Short Circuit

- A short circuit means the voltage is zero \((v = 0)\).
- Current through a short circuit can be any value.
- A short circuit is equivalent to a resistance of \(0 \, \Omega\).
- Short circuit summary:
  - Zero resistance;
  - Zero voltage drop;
  - Current can be any value.
Resistance (R) and Conductance (G)

- **Resistance** (R) is the capacity of a material to impede the flow of current.
- More current will flow if there is less resistance.
- **Conductance** (G) is the inverse of resistance.
- The *unit* of resistance is the ohm (R), has the symbol Ω, and has units of (volts/amp).
- The circuit symbol used for a resistor of R ohms is:

![Resistor Symbol](image)

Ohm’s Law

- The relationship between voltage, current, and resistance is defined by Ohm’s law which states that

\[ V = IR \]

where

- \( V \) = the voltage in volts;
- \( I \) = the current in amps;
- \( R \) = the resistance in ohms.

![Ohm's Law Diagram](image)
Other Forms of Ohm’s Law

- Ohm’s law can be expressed in any one of three forms, depending on which quantities are known:

  \[ V = IR \quad \text{(I and R known)} \]
  \[ I = \frac{V}{R} \quad \text{(V and R known)} \]
  \[ R = \frac{V}{I} \quad \text{(V and I known)} \]

Ohm’s Law Illustrated
Ohm’s Law Examples

Using Ohm’s law, calculate the values of $v$ and $i$ in the examples below and determine the power dissipated in each resistor.

![Ohm's Law Diagrams](image)

a) $v_a = 8V$  $p = 8W$

b) $i_b = 10A$  $p = 500W$

c) $v_c = -20V$  $p = 20W$ (positive)

d) $i_d = -2A$  $p = 100W$

Wire Gauge and Resistivity

- The resistance of a wire is determined by the resistivity of the conductor as well as the geometry:

$$R = \rho \frac{l}{A}$$

- In most cases, the resistance of wires can be assumed to be 0 $\Omega$. 

Cross-sectional area = $A$ cm$^2$

Direction of current flow

Resistivity = $\rho$ $\Omega$·cm
Resistors

Resistor Color Code Chart

<table>
<thead>
<tr>
<th>COLOR</th>
<th>1st BAND</th>
<th>2nd BAND</th>
<th>3rd BAND</th>
<th>MULTIPLIER</th>
<th>TOLERANCE</th>
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<td>0</td>
<td>1Ω</td>
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<tr>
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<td>1</td>
<td>10Ω</td>
<td>± 1% (F)</td>
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<tr>
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<td>1</td>
<td>100Ω</td>
<td>± 2% (G)</td>
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<td>3</td>
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<td>± 1% (F)</td>
</tr>
<tr>
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<td>4</td>
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<td>± 2% (G)</td>
</tr>
<tr>
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<td>5</td>
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<td>100kΩ</td>
<td>± 0.5% (D)</td>
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<tr>
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<td>± 0.25% (C)</td>
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<td>± 10% (B)</td>
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Ohm’s Law Example

Calculate the value of R from the information shown in the graph below:

\[
R = \frac{V}{I} = \frac{8\text{V}}{4\text{A}} = 2\ \Omega
\]

R is the inverse of the slope of the line:

\[
R = \frac{V}{I} = \frac{(8\text{V})}{(4\text{A})} = 2\\Omega
\]

Power and its Alternate Forms

• Resistors always absorb power:

\[
p = vi = \frac{v^2}{R} = i^2R
\]
Power Example

Calculate the power absorbed or generated by both elements in the circuit below:

- Resistor absorbs power: $10V \times 2.5mA = 25mW$
- DC source generates power: $10V \times -2.5mA = -25mW$
- Sum of all power in a circuit must = 0

*Note: Resistors always absorb power but DC sources can either generate or absorb power.*

Textbook Problem 2.9 (Nilsson 11E)

Find the total power developed in the circuit if $v_o = 5$ V.

$P_{gA} = -135W$  $P_{20V} = 180W$  $P_{10va} = -75W$
$P_{vg} = 27W$  $P_{6A} = 3W$
$P_{total} = 210W$ and generated power = absorbed power
Nodes, Paths, Loops, Branches

- These two circuits are equivalent.
- There are three nodes and five branches:
  - Node: a point at which two or more elements have a common connection;
  - Path: a sequence of nodes;
  - Branch: a single path in a circuit composed of one simple element and the node at each end of that element;
  - Loop: a closed path.

Kirchhoff’s Current Law

- Kirchhoff’s Current Law (KCL) states that the algebraic sum of all currents entering a node is zero.

\[
i_A + i_B + (-i_C) + (-i_D) = 0
\]
KCL: Alternative Forms

- Current \( IN \) is positive:
  \[ i_A + i_B + (-i_C) + (-i_D) = 0 \]

- Current \( OUT \) is positive:
  \[ (-i_A) + (-i_B) + i_C + i_D = 0 \]

- Current \( IN = \) Current \( OUT \):
  \[ i_A + i_B = i_C + i_D \]

KCL Application

Find the current through resistor \( R_3 \) if it is known that the voltage source supplies a current of 3 A.

Answer: \( i = 6 \text{ A} \)
Kirchhoff’s Voltage Law

- Kirchhoff’s Voltage Law (KVL) states that the algebraic sum of the voltages around any closed path is zero.

\[-v_1 + v_2 + -v_3 = 0\]

KVL: Alternative Forms

- Sum of \textit{RISES} is zero (clockwise from B):
  \[v_1 + (-v_2) + v_3 = 0\]
- Sum of \textit{DROPS} is zero (clockwise from B):
  \[(-v_1) + v_2 + (-v_3) = 0\]
- Sum of \textit{RISES} is equal to sum of \textit{DROPS} (clockwise from B):
  \[v_1 + v_2 = v_3\]
KVL Application

Find the current $i_x$ and the voltage $v_x$

Answer: $v_x = 12\, V$ and $i_x = 120\, mA$

Textbook Problem 2.18 (Nilsson 10E)

Find the values of $i_a$, $i_b$, $v_o$, and the power absorbed or generated by all circuit elements.

$i_a = 2A$  
$i_b = 0.5A$  
$v_o = 40V$  
$P_4 = 25W$  
$P_{20} = 80W$  
$P_{80} = 20W$  
$P_{50v} = -125W$ (delivered)
Circuit Analysis with Dependent Sources

• Circuits that contain dependent sources can be analyzed using Ohm’s and Kirchhoff’s laws.
• A dependent source generally adds another equation to the solution process.

Textbook Example Figure 2.22 (Nilsson 11E)

A) Use Kirchhoff’s and Ohm’s laws to find the voltage $v_o$
B) Show that the total power developed equals the total power dissipated.

A) $v_o = 480 \text{ V}$
B) Power developed = - 21.7 W
Textbook Problem 2.22 (Nilsson 10E)

The current $i_0$ is 1 A.

A) Find $i_1$. 
B) Find the power dissipated in each resistor. 
C) Verify that the power developed = power absorbed.

\[ A) \quad i_1 = 2A \]
\[ B) \quad P_4 = 100W \quad P_{50} = 50W \quad P_{10} = 90W \quad P_{65} = 260W \quad P_{25} = 400W \]
\[ C) \quad P_{150V} = 900W = \text{sum of powers dissipated in the 5 resistors.} \]

Chapter 2 Summary

- Understand symbols and behavior of the following circuit elements:
  - Independent voltage and current sources;
  - Dependent voltage and current sources;
  - Resistors.
- Defined and used Ohm’s and Kirchhoff’s laws to analyze circuits.
- Illustrated how to calculate power for each element in a circuit and determine whether or not power balances.