Thévenin and Norton Equivalent Circuits

Name ________________________________

Introduction
Solving electrical networks is often made easier by using Thévenin and Norton equivalent circuits to model an entire network. The Thévenin and Norton equivalents as shown below are themselves equivalent to each other as long as \( V_{TH} = I_N R_{TH} \). This experiment considers how to determine the parameters of the equivalent circuits for a T-configuration of resistors attached to a voltage source as shown in step 2 on the following page.

\[ R_{TH} \]
\[ V_{TH} \]
\[ I_N \]

Thévenin Equivalent

Norton Equivalent

Objectives
- Verify Thévenin’s theorem;
- Verify Norton’s theorem;
- Verify the Maximum Power Transfer theorem.

Equipment
- Digilent Analog Discovery 2 Module;
- Handheld DMM;
- Breadboard;
- Assorted resistors;
- Assorted wires.

References
- Zybooks text book;
- Course web site;
- Resistor color-code chart.
**Note:** For any measured value using your multi-meter, record the value displayed. For calculations, round your results to four significant digits but carry more digits in intermediate calculations.

**Procedure**

1) Select three of your resistors with the values shown below. Measure and record their resistance values. Calculate percentage error using the following formula:

\[
\text{Percentage Error} = \frac{|(\text{Nominal value} - \text{Measured value})|}{(\text{Nominal value})} \times 100\%
\]

<table>
<thead>
<tr>
<th>Resistor</th>
<th>Nominal Value</th>
<th>Measured Value</th>
<th>Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>100 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₂</td>
<td>560 Ω</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td>330 Ω</td>
<td></td>
<td></td>
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</tbody>
</table>

2) Using the resistor values you measured in step 1, calculate the Thévenin resistance \(R_{TH}\) in the circuit below and record it in step 3. This is the resistance seen looking into the circuit from the right, i.e. into the + and – terminals. Remember to replace the voltage source with a short circuit.

3) Wire up the T-network circuit shown above, except initially replace the voltage source with a short circuit and measure the Thévenin resistance \(R_{TH}\) with your meter. This is the resistance seen looking into the circuit from the right, i.e. into the + and – terminals. Record the measured value and calculate the % Error.

\[
R_{TH, \text{Calculated}} = \text{_________} \text{Ω} \quad R_{TH, \text{Measured}} = \text{_________} \text{Ω} \quad \text{Error} = \text{_________} \%
\]
4) Connect the DC voltage source ($V$) in the network above and set its value to 5.00 volts. Calculate (think voltage divider) and measure the open-circuit voltage $V_{TH}$ at the output.

$$V_{TH, \text{Calculated}} = \underline{\text{__________}}\ \text{V} \quad V_{TH, \text{Measured}} = \underline{\text{__________}}\ \text{V} \quad \text{Error} = \underline{\text{__________}}\ %$$

5) Short the + and – terminals at the output. Calculate and measure the short-circuit current (Norton current) flowing from the + to the – terminals.

$$I_{N, \text{Calculated}} = \underline{\text{__________}}\ \text{mA} \quad I_{N, \text{Measured}} = \underline{\text{__________}}\ \text{mA} \quad \text{Error} = \underline{\text{__________}}\ %$$

6) Replace your circuit in step 2 with its Thévenin equivalent (they’re identical, right?). You will need to use the resistor in your inventory that is closest to the Thévenin resistance. Now, one at a time, place a resistor R across the + and – terminals at the output of your Thévenin equivalent circuit. Be creative in coming up with the correct resistance values that are close to the values in the table below. You don’t need an exact match – ball park is fine. Note that you will have to combine some of your resistors in parallel, some in series, and maybe even a combination. Calculate and measure the voltage across the resistor ($V_R$) and the current through the resistor ($I_R$).

<table>
<thead>
<tr>
<th>R Nominal</th>
<th>30</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>700</th>
<th>800</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{Meas}$ (Ω)</td>
<td></td>
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<tr>
<td>$V_{R, \text{Calc}}$ (V)</td>
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</tr>
<tr>
<td>$V_{R, \text{Meas}}$ (V)</td>
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<tr>
<td>$I_{R, \text{Calc}}$ (mA)</td>
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<td></td>
</tr>
<tr>
<td>$I_{R, \text{Meas}}$ (mA)</td>
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</tr>
</tbody>
</table>
7) Using Excel or a similar graphing program, do a least squared error trendline fit to the calculated \((I_R, V_R)\) data in the table above to determine the calculated Thévenin voltage, \(V_{TH}\), and the calculated Norton current, \(I_N\). Plot \(V_R\) (V) on the y-axis and \(I_R\) (mA) on the x-axis. The Thévenin voltage is the y-intercept, i.e. the open-circuit voltage, and the Norton current is the x-intercept, i.e. the short-circuit current. The slope of the line is the negative of the Thévenin (Norton) resistance divided by 1000. Label these points on the graph and record their values below, along with the calculated value of \(R_{TH}\). Print out a copy of the graph.

\[
\begin{align*}
V_{TH} &= \phantom{-} _____ \text{ V} \\
I_N &= \phantom{-} _____ \text{ mA} \\
R_{TH} &= \phantom{-} _____ \text{ } \Omega \\
r^2 &= \phantom{-} _____ \\
\end{align*}
\]

8) Next, do a least squared error trendline fit to the measured \((I_R, V_R)\) data in the table above to determine the Thévenin voltage, \(V_{TH}\), and the Norton current, \(I_N\). Label these points on the graph and record their values below, along with the calculated value of \(R_{TH}\). Print out a copy of the graph.

\[
\begin{align*}
V_{TH} &= \phantom{-} _____ \text{ V} \\
I_N &= \phantom{-} _____ \text{ mA} \\
R_{TH} &= \phantom{-} _____ \text{ } \Omega \\
r^2 &= \phantom{-} _____ \\
\end{align*}
\]

9) Compile all measured and calculated values in the table below.

<table>
<thead>
<tr>
<th></th>
<th>(V_{TH})</th>
<th>(I_N)</th>
<th>(R_{TH})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Calculated</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>TrendlineCalc (step 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TrendlineMeas (step 8)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10) Using values of $V_{TH}$ and $R_{TH}$ from the trendline analysis, and the equation below, calculate the power dissipated in the resistor $R$ for all values of $R$ shown in the table. Compute $P_R$ from both your calculated and measured data.

$$P = \left( \frac{V_{TH}}{R_{TH} + R} \right)^2 R$$

| $R \ \Omega$ | 51||100 | 51 | 100 | 330||470 | 330 | 470 | 560 | 667 | 767 | 1000 |
|--------------|----------------|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $P_{Calc}$ (mW) |               |      |     |     |     |     |     |     |     |     |     |     |     |
| $P_{Meas}$ (mW) |               |      |     |     |     |     |     |     |     |     |     |     |     |

11) Using Excel, plot the calculated and measured power from step 10 above versus the load resistance $R$. Place power on the y-axis and resistance on the x-axis. Use different symbols for each and identify the curves as calculated and measured. Print out a copy of the graph.

12) From the Thévenin equivalent circuit (using the values of $V_{TH}$ and $R_{TH}$ from the measured trendline analysis in step 9) and the Maximum Power Theorem, calculate the maximum power dissipated by $R$ and the value of $R$ when this occurs. Compare this result with what you observe from the plot in step 11.

<table>
<thead>
<tr>
<th>$P_{\text{MAX}}$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{MAX}}$ Theorem</td>
<td></td>
</tr>
<tr>
<td>Curve of calculated power (eyeball from graph)</td>
<td></td>
</tr>
<tr>
<td>Curve of measured power (eyeball from graph)</td>
<td></td>
</tr>
</tbody>
</table>

To Turn In
Do your work on either the .pdf or .docx version of the lab and place in the lab3 drop box by 11:59pm on Sunday, April 26.