Twenty-Third Annual
WWU University Days
MATHEMATICS COMPETITION
2010

Winners — Past Five Years		
	1st	**2nd**
2005	AAA	MAA
2006	WWVA	PAA
2007	CAA	UCA
2008	WWVA	UCA
2009	UCA	MAA

team member(s)	head math teacher	academy/h.s.

This is a *TEAM* competition. Your answers will be graded based both on **CORRECTNESS** and on the **QUALITY** of your presentation. Show your work *neatly* and indicate your answers *clearly.* Answers *without* justification will receive *no* credit. Please place *no more than* one (1) solution per page. When you are done, arrange your team solutions in numerical order (1, 2, 3, ..., 10) before you hand them in. Good luck!!

1. Find the number of digits in $32^{16}125^{25}$ (when written in the usual base 10 form).

2. A calculator has a key which replaces the displayed entry with its square, and another key which replaces the displayed entry with its reciprocal. Let y be the final result if one starts with an entry $x \neq 0$ and alternately squares and reciprocates n times each. Assuming the calculator is completely accurate (e.g, no roundoff or overflow), then y equals

   (a) $x^{((-2)^n)}$ (b) x^{2n} (c) x^{-2n} (d) $x^{-(2n)}$ (e) $x^{((-1)^n 2n)}$

3. Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N?

4. A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of the circle?

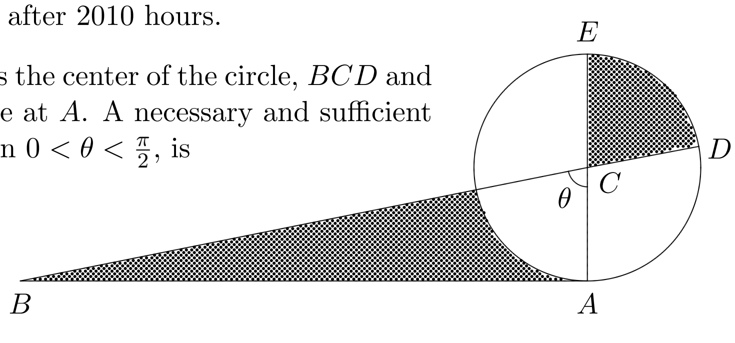
5. What is the smallest positive integer that can be expressed as the sum of two not-necessarily distinct squares of integers? Of course, justify why your number is the smallest.

6. Pick any two consecutive integers. Add them together. Add 9 to the sum. Divide the sum by 2, then subtract the smaller of the two original numbers. Prove that no matter what two numbers are initially chosen, you will always get the same result.

7. "Lucky Zeno," the turtle, was trying to cross a busy road. Full of energy, he crossed half-way in one hour and was very thankful to be alive. During the next hour he was tiring and was only able to cover half the remaining distance. He kept tiring and again was only able to cover half the remaining distance during the third hour. If this "halving" continues, express precisely, as a fraction using only the digits 0, 1, and 2 (multiple times if necessary), the remaining distance Zeno needs to cover after 2010 hours.

8. In the configuration below, θ is measured in radians, C is the center of the circle, BCD and ACE are line segments, and AB is tangent to the circle at A. A necessary and sufficient condition for the equality of the two shaded areas, given $0 < \theta < \frac{\pi}{2}$, is

   (a) $\tan\theta = \theta$ (b) $\tan\theta = 2\theta$
   (c) $\tan\theta = 4\theta$ (d) $\tan 2\theta = \theta$
   (e) $\tan\frac{\theta}{2} = \theta$



9. What is the largest integer n for which $n^{200} < 5^{300}$?

10. Find $\frac{1000^4}{(252^2-248^2)^2}$.

Solutions:

1. $32^{16}125^{25} = (2^5)^{16} \cdot (5^3)^{25} = 2^{80}5^{75} = 2^5 \cdot 10^{75}$ which has 77 digits.
2. (a)
3. $T = A + B + C$ and $T - N = 2([A - N] + [B - N] + [C - N])$. Solve and get $T/N = 5$.
4. See last page.
5. $1 = 0^2 + 1^2$.
6. $(N + N + 1 + 9)/2 - N = 5$.
7. $1/\left(2^{2010}\right)$.
8. (b)
9. 11. $5^{300} = (5\sqrt{5})^{200}$ and $11^2 = 121 < 125 = (5\sqrt{5})^2$ but $12^2 = 144$.
10. $\frac{1000^4}{(252^2-248^2)^2} = \left(\frac{1000^2}{252^2-248^2}\right)^2 = \left(\frac{1000^2}{(252+248)(252-248)}\right)^2 = \left(\frac{1000^2}{500\cdot 4}\right)^2 = 500^2 = 250,000.$