Twenty-Second Annual
WWU University Days
Mathematics Competition
2009

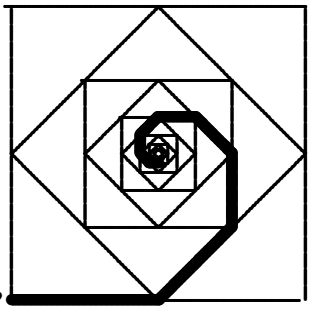
Winners — Past Five Years		
	1st	2nd
2004	Tie: AAA & WWVA	
2005	AAA	MAA
2006	WWVA	PAA
2007	CAA	UCA
2008	WWVA	UCA

team member(s)	head math teacher	academy/h.s.

This is a *TEAM* competition. Your answers will be graded based both on **CORRECTNESS** and on the **QUALITY** of your presentation. Show your work *neatly* and indicate your answers *clearly*. Answers *without* justification will receive *no* credit. Please place *no more than* one (1) solution per page. When you are done, arrange your team solutions in numerical order (1, 2, 3, ..., 10) before you hand them in. Good luck!!

1. Fisherman Tyler saves lead for large (50 pound) spherical trolling-line weights. Troller Charlie has a mold for pouring such weights. Tyler has 2,009 pounds of lead saved and wants Charlie to pour weights for him with the following stipulations: Charlie charges Tyler \$0.30 per pound to pour the weights, Charlie will buy any amount of lead Tyler will sell for \$0.50 per pound, and Tyler wants as many weights he can get from the 2,009 pounds *without* paying to or receiving any cash from Charlie. Exactly how many 50 pound weights can Tyler get? Is there any lead left over?

2. Consider the infinite set of nested squares on the right. A path starts at point P and moves one-half edge length before making a 45° left-hand turn and continues doing this *ad infinitum*. If the original square has edge length 1 unit, how long is the path?



3. Is $\dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}$ irrational? Explain.

4. Verify the trigonometric identity: $\ln|\sec x| = -\ln|\cos x|$.

5. A class of 100 students is having an election for class president. Greg, Marcia, and Peter are the candidates, and everyone, including the candidates, is required to vote in the election. After the election, the vote tallies will be announced for all three candidates. Note that a 3-way tie is not possible. However, if two candidates tie for the most votes, then there will be a runoff between them. How many different possible tallies will lead to a runoff?

6. Six distinct positive integers between 1 and 2009 inclusive are chosen at random. What is the probability that some pair of these integers has a difference that is a multiple of 5?

7. The positive integers A, B, $A-B$, and $A+B$ are all primes less than 1000. What is the sum of these four primes?

8. A county park is in the shape of a regular hexagon $2\,km$ on a side. Starting at a corner, Alice walks along the perimeter of the park for a distance of $5\,km$. How many kilometers is she from her starting point?

9. Find the exact value of: $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$.

10. A wooden cube with edge length n units (where n is an integer greater than 2) is painted black all over. By slices parallel to its faces, the cube is cut into n^3 smaller cubes each of unit edge length. If the number of smaller cubes with just one face painted black is equal to the number of smaller cubes completely free of paint, what is n?

Solutions:

1. 25 weights with 9 pounds left over. Note that $(25)(50) = 1250$. Then, $1250 \cdot 0.30 = 375$ and $750 \cdot 0.50 = 375$. Finally, $2000 - (1250 + 750) = 9$.
2. Geometric series: $\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + \cdots = \frac{1/2}{1 - 1/\sqrt{2}} = \frac{2 + \sqrt{2}}{2}$.
3. Yes. It is equal to 2. Just square the expression and get 4.
4. Very easy.
5. $51 = 3 \cdot 17$.
6. 1/5. There are 20 possible differences. Look at the last digit.
7. 17. Note that $A - B$ and $A + B$ must both be odd so that $B = 2$. But then $A - B$, A, and $A + B$ are 3 consecutive odd numbers and prime. So $A = 3$ and the sum is $3 + 2 + 5 + 7 = 17$.
8. $\sqrt{13}$. Easy geometry: $\sqrt{(\sqrt{3} + \sqrt{3})^2 + 1^2} = \sqrt{13}$.
9. 3. $x = \sqrt{6 + x} \Rightarrow x^2 = 6 + x \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x + 2)(x - 3) = 0 \Rightarrow x = 3$. $x = -2$ is extraneous.
10. 8. Painted one side: $6(n - 2)^2$. Unpainted: $(n - 2)^3$. So, $6(n - 2)^2 = (n - 2)^3 \Rightarrow 6 = n - 2 \Rightarrow n = 8$.